The planar rook monoid P_n consists of specially defined bipartite graphs together with an operation called "diagram stacking." Taking the complex span of the elements of P_n, we obtain the planar rook algebra $\mathbb{C}P_n$. The goal of the first half of this Independent Study is to show that $\mathbb{C}P_n$ decomposes as the direct sum of certain special subspaces.

The subspaces we are concerned with are related to the action of P_n on $\mathbb{C}P_n$ via left multiplication. When a subspace V of $\mathbb{C}P_n$ is closed under the action of P_n, we say that V is P_n-invariant. When V does not contain any proper nontrivial P_n-invariant subspaces, we say that V is irreducible. That is, we are looking for subspaces that are closed under the action of P_n and are, in some sense, the "smallest" ones closed under the action of P_n. We find these subspaces and show that $\mathbb{C}P_n$ can be expressed as the direct sum of them. When such an expression is possible, we call the algebra in question semisimple.

The second half of this Independent Study looks at coloring $\mathbb{C}P_n$. We first take a "palette" group G and color the edges of diagrams in P_n by assigning group elements to them. This creates $P_n(G)$, a monoid consisting of multiple copies of the elements of P_n (one for each possible color scheme) and an updated diagram multiplication to account for the operation of G. Taking the complex span of the colored diagrams, we obtain the colored planar rook algebra $\mathbb{C}P_n(G)$.

The colored planar rook algebra's semisimplicity depends on the palette group. For finite abelian G, we explore why $\mathbb{C}P_n(G)$ is semisimple. We also see, however, that $\mathbb{C}P_1(S_3)$ is not semisimple, the proof for this fact exemplifying that there exist too few $P_1(S_3)$-invariant irreducible subspaces for the proper decomposition.

The only example of a non-semisimple colored planar rook algebra that we have is $\mathbb{C}P_1(S_3)$. The most enticing next step in this research is to examine $\mathbb{C}P_n(G)$ for other non-abelian groups, the next smallest of which (after S_3) are Q_8 and D_4.