Exoplanets Sunsets

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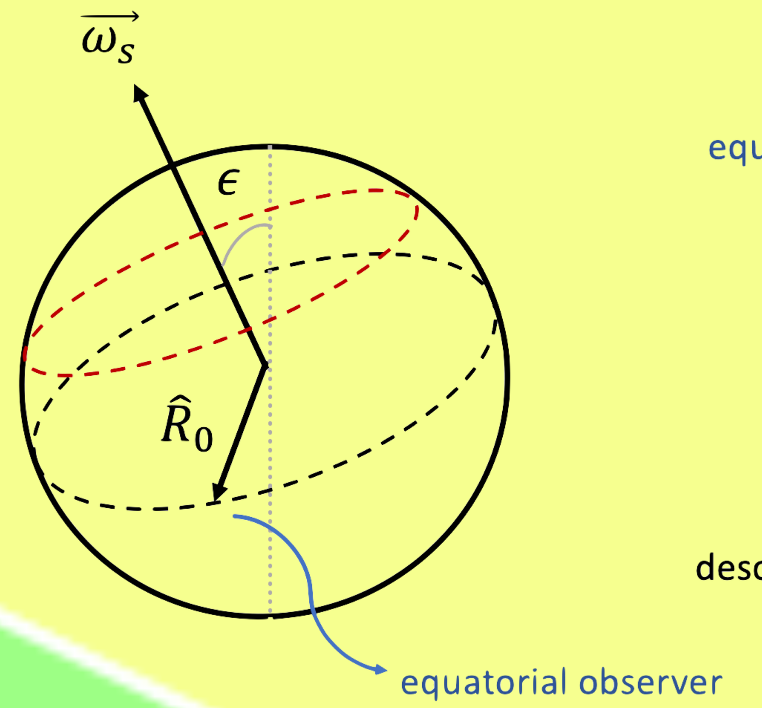
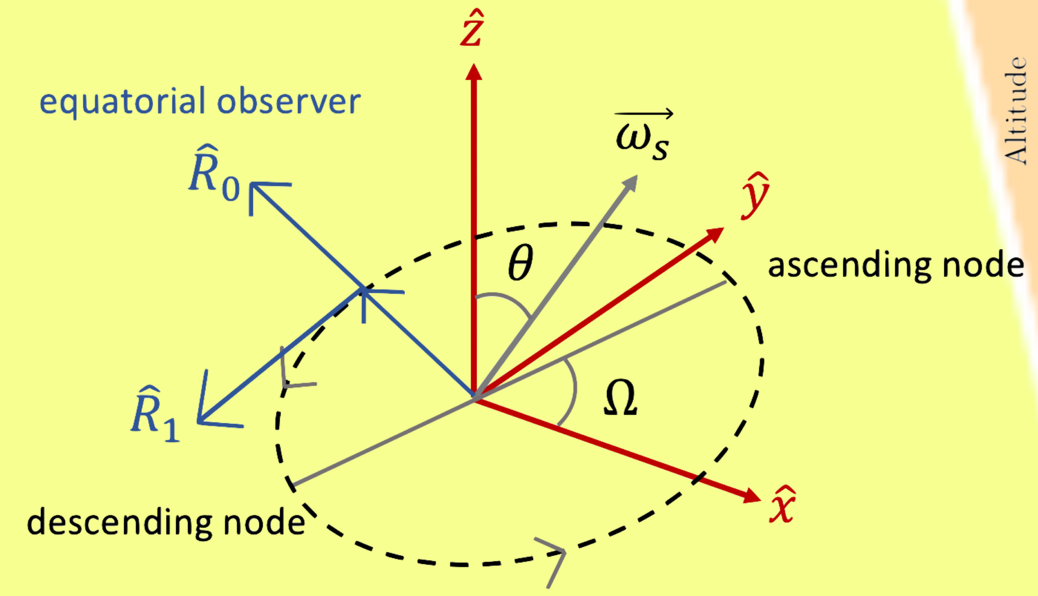
3D Model

(Obliquity $\epsilon \neq 0$)

Planet Frame



Host Star Frame



$\epsilon = \theta$

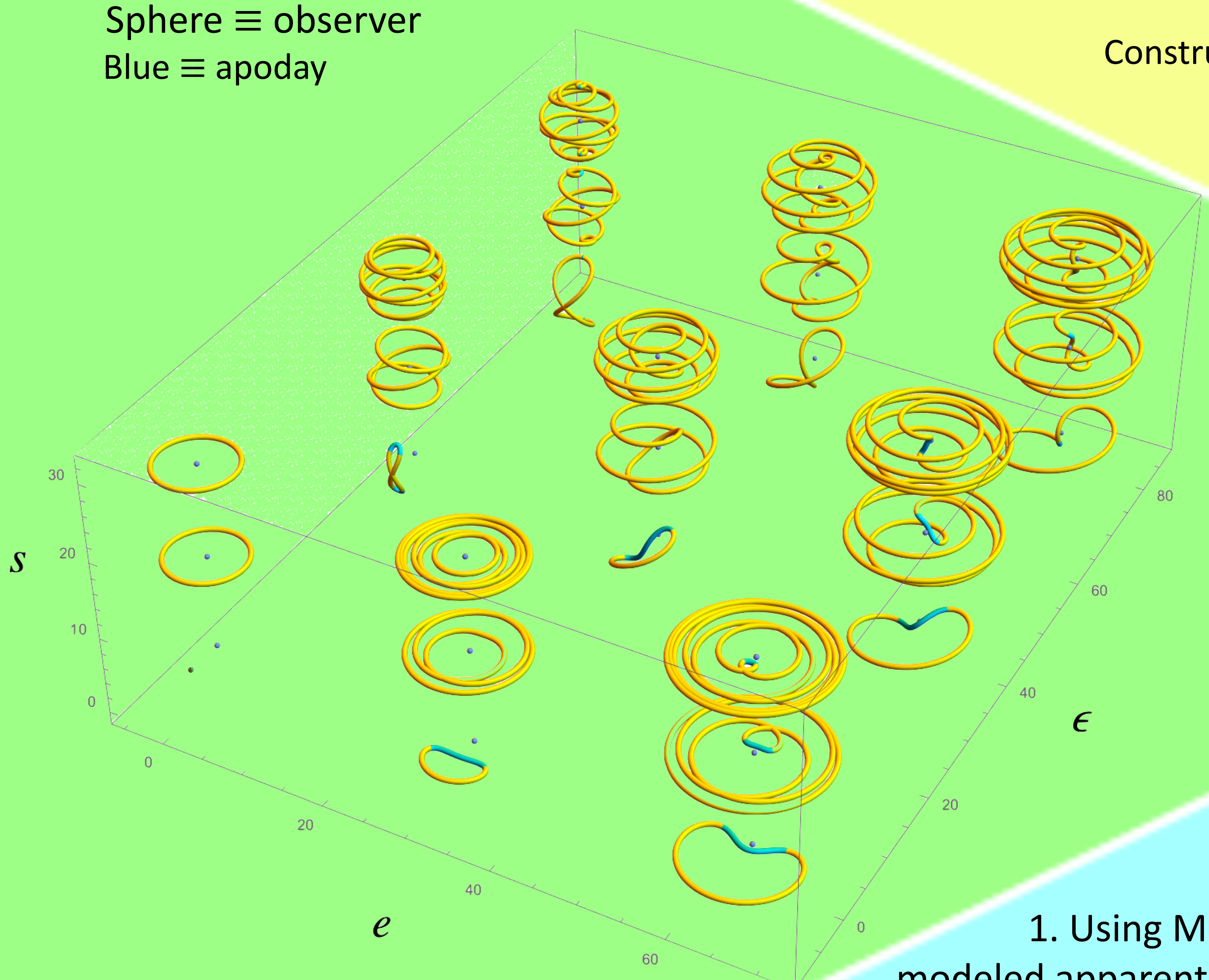
Construct planet frame $\{\hat{R}_0, \hat{R}_1, \hat{\omega}_s\}$

$\hat{R}_1 = \hat{R}_0 \times \hat{\omega}_s$

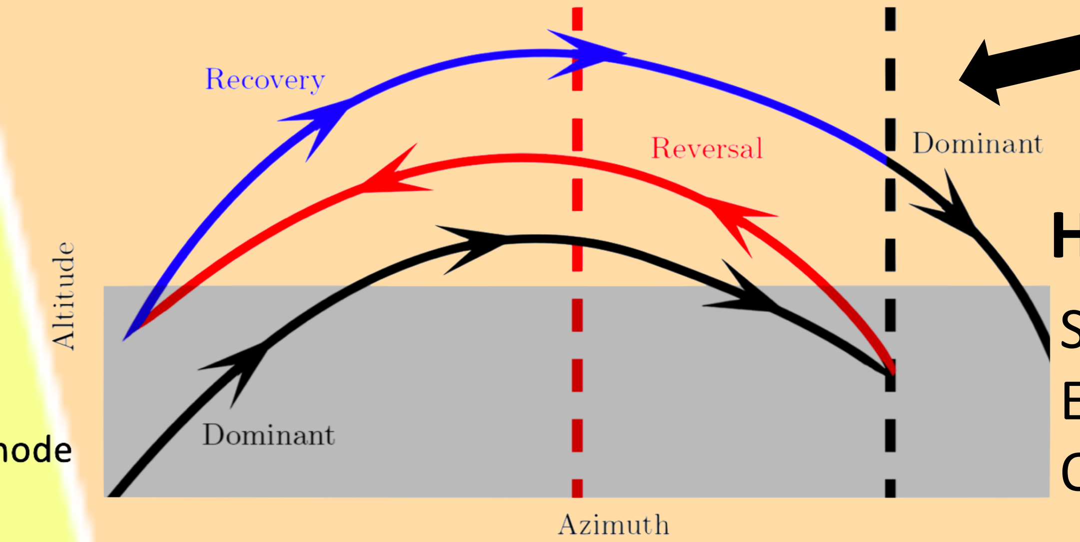
Results:

Sphere ≡ observer

Blue ≡ apoday



Apoday



Hypothesis:

Spin-orbit ratio

Eccentricity

Obliquity

Apoday:

The host star appears to move backwards in the sky observed from planet.

Goal:

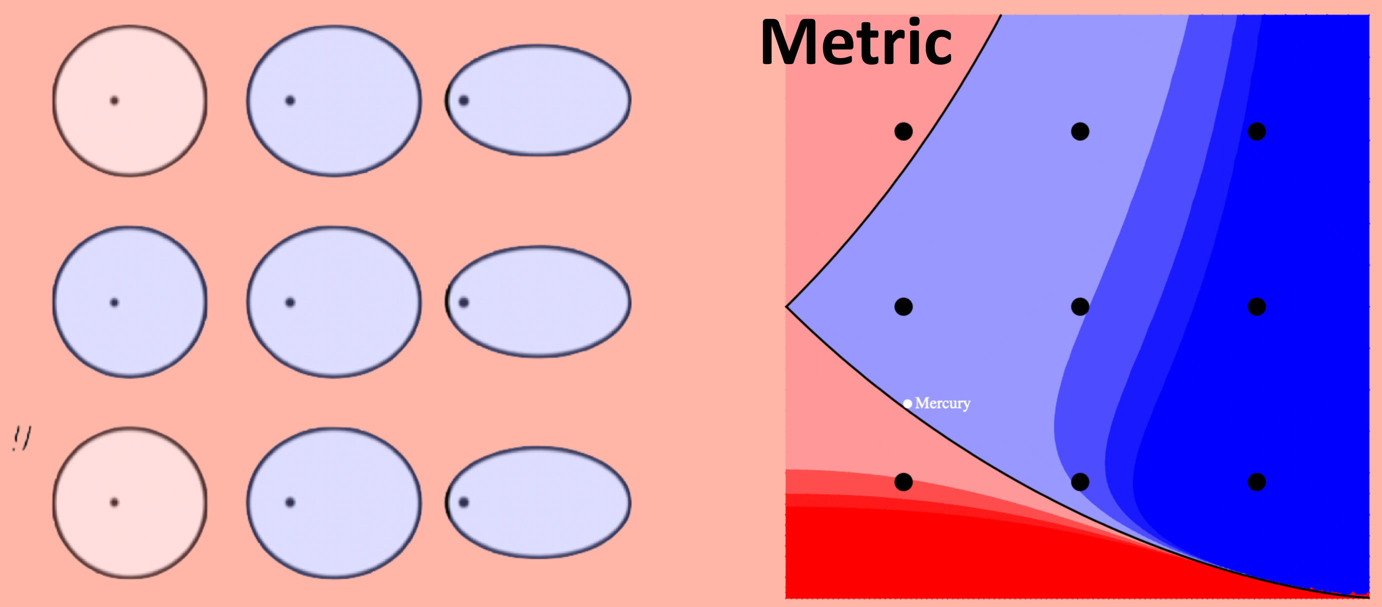
Find the necessary condition for apoday to happen

2D Model (Obliquity $\epsilon = 0$)

Eccentricity & Spin-orbit ratio → Determine apoday

Blue apoday

Red no apoday



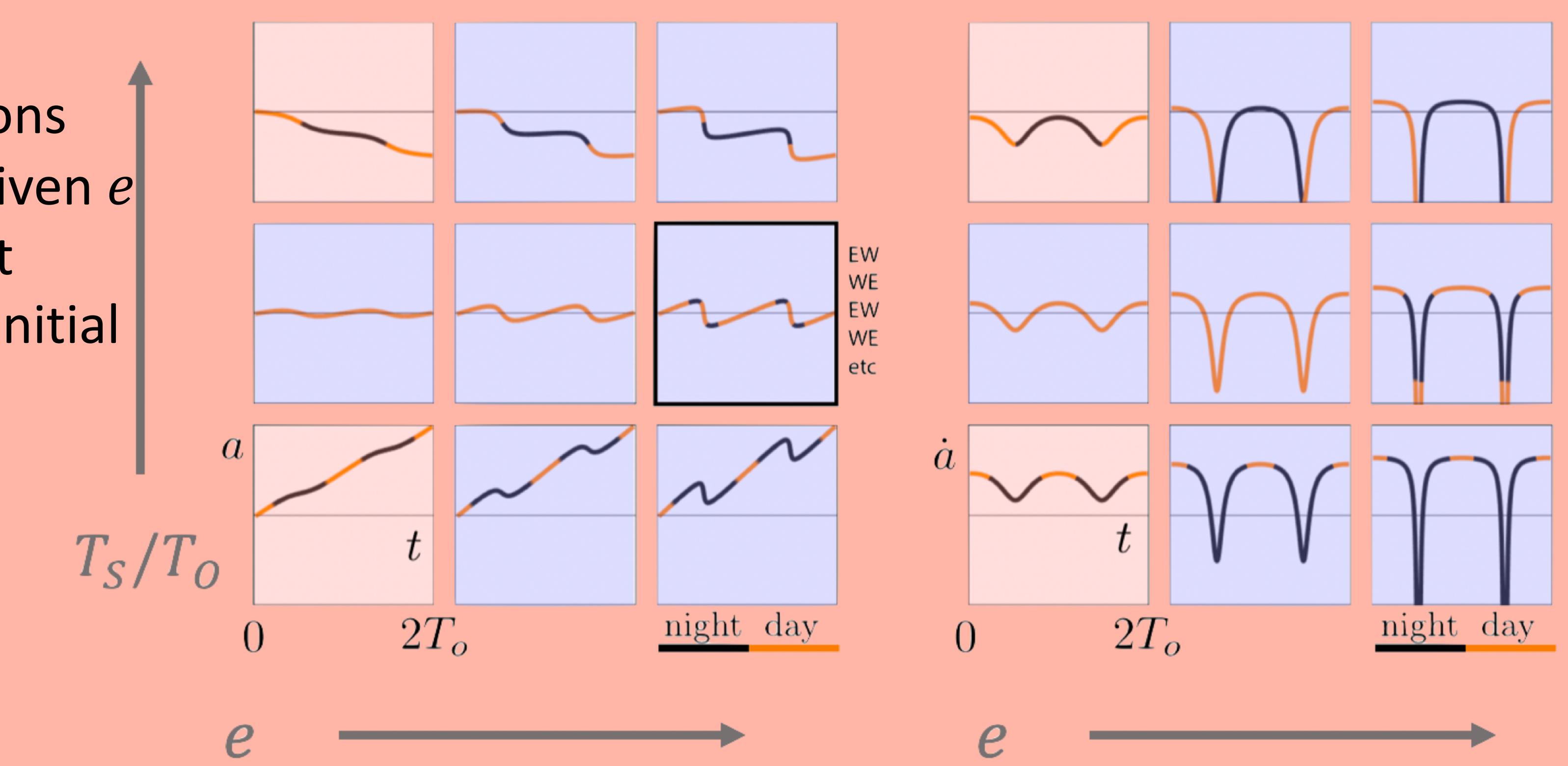
Input:

Eccentricity $\equiv e$

Spin-orbit ratio $\equiv T_s/T_o$

Steps:

1. Set up differential equations using Kepler's La ≡ w so that given e and T_s/T_o we get the planet orbit model. Let Mathematica solve the initial value problem numerically.
2. Plot the orbit, altitude of the host star, and define a metric to determine apoday .
3. Make a grid of the plots.



3D

2D

Find Boundaries

$\omega_s \equiv$ planet spin angular velocity

$\omega_a \equiv$ planet orbital angular velocity at aphelion

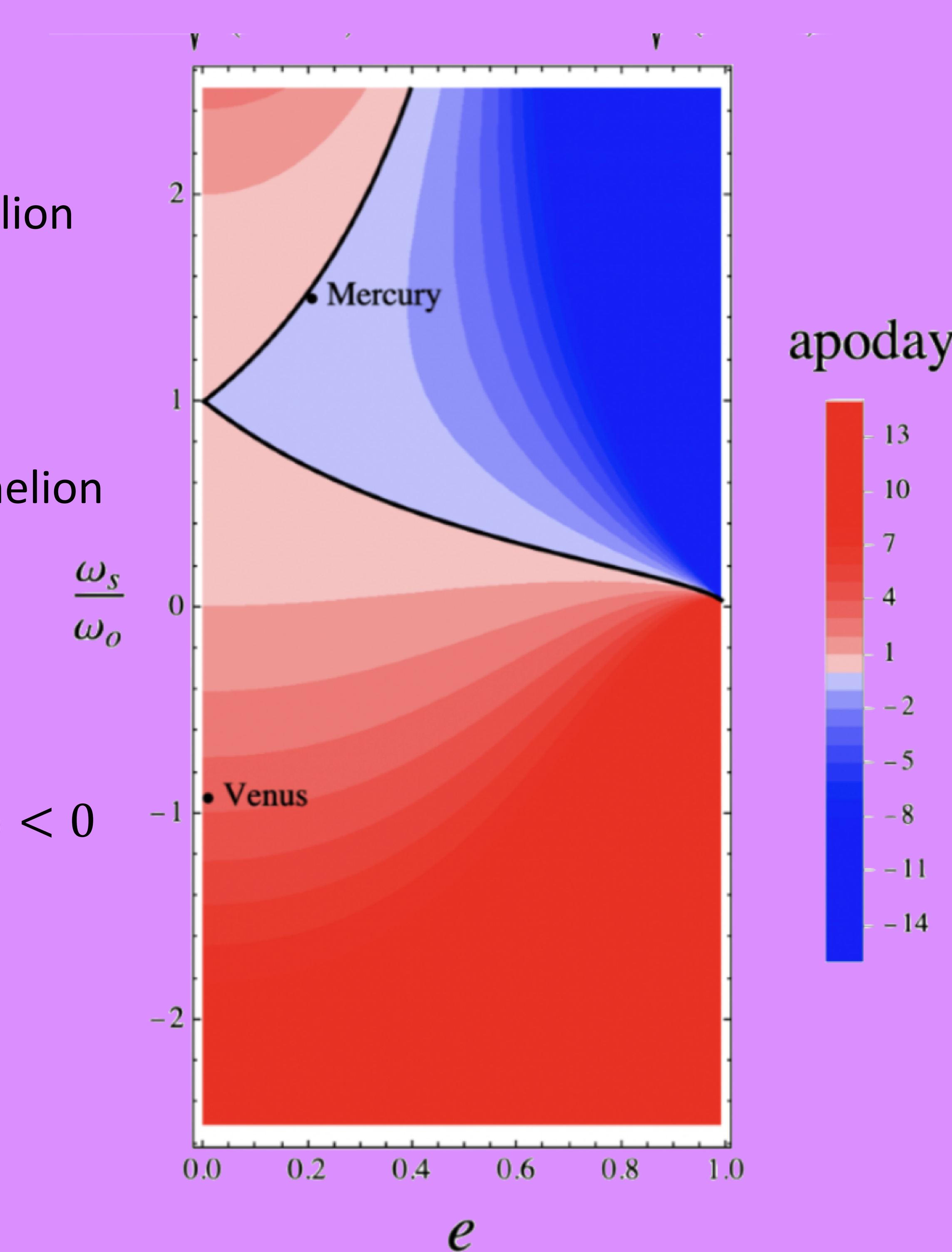
$$\omega_a = \frac{2\pi}{T_O}\sqrt{\frac{1-e}{(1-e)^3}}$$

$\omega_p \equiv$ planet orbital angular velocity at perihelion

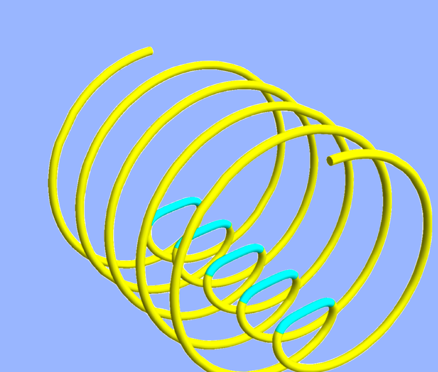
$$\omega_p = \frac{2\pi}{T_O}\sqrt{\frac{1+e}{(1-e)^3}}$$

$$(\omega_s - \omega_a)(\omega_s - \omega_p) < 0$$

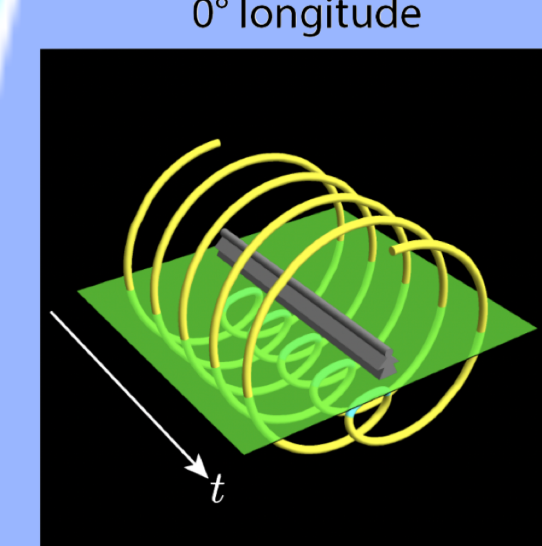
Apoday



Helix Plot

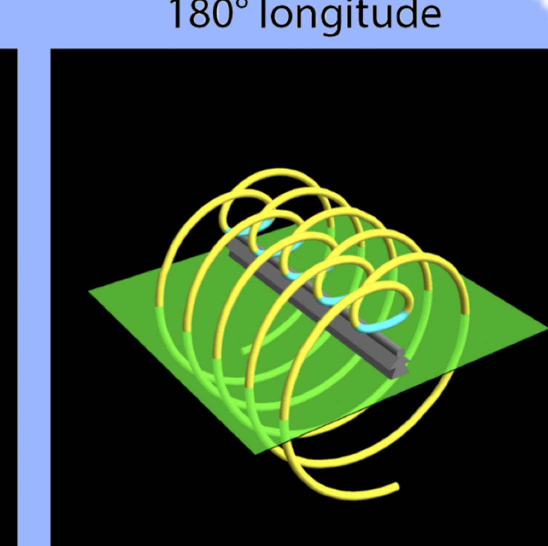


0° longitude



apodays unobserved

180° longitude



apodays observed

Conclusion

1. Using Mathematica simulations, we modeled apparent host star motion in exoplanet for different orbital eccentricity, spin-orbit ratio and obliquity.
2. For the special case of zero obliquity(tilt), an exact nonlinear equations delimiting apodays in the space of orbital eccentricity and spin-orbit (day-year) ratio is derived, confirmed by numerical simulations.

Acknowledgement

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