Alien Sunsets on Tumbling Asteroids

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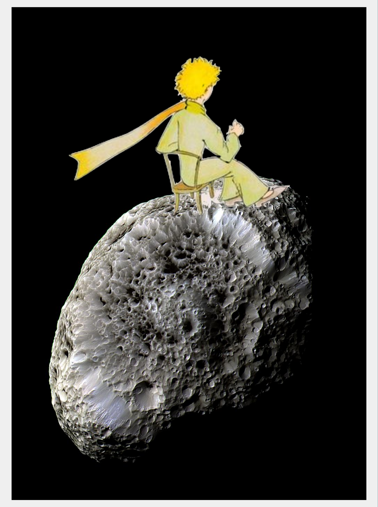
Abstract

We simulated Sol's apparent motion as observed from tumbling asteroids. If all the mass of the asteroid is concentrated at a point, it will move in an elliptical orbit, like Earth. To account for the tumbling of the asteroid, we extend the point into a dumbbell. We forced the center of mass to move elliptically, and then the asteroid itself tumbled, according to Newton's second law.

We calculated Sol's apparent motion by numerical integration of Kepler and dumbbell's orbital equations and changeable parameters. Our system is very sensitive to initial conditions, and hence is chaotic.

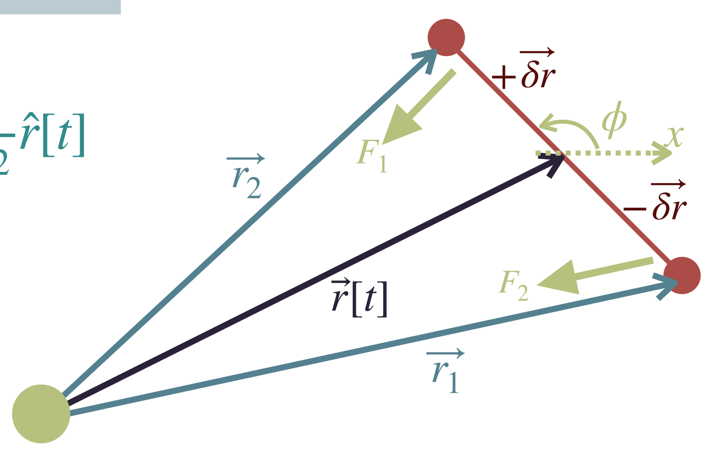
We simulated Sol's movement in skies under different conditions from observer's perspective. Sol is able to move irregularly in the skies; it may move back and forth in the sky or stay below the horizon for days.



2D Single Dumbbell

Cartesian Coordinate

$$\begin{cases} \ddot{\vec{r}}[t] = \dfrac{-GM}{x[t]^2+y[t]^2+z[t]^2}\hat{r}[t] \\ \dot{\vec{r}}[0] = \{0, v_0, 0\} \\ \vec{r}[0] = \{x_0, 0, 0\} \end{cases}$$

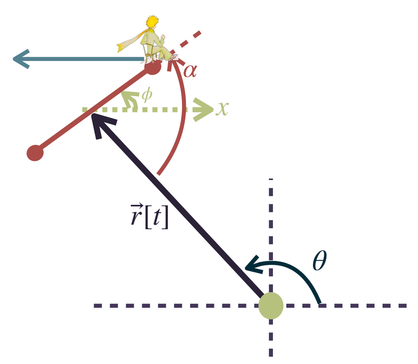


$$\begin{cases} I\,\ddot{\vec{\phi}}[t] = \{0,0,1\}\cdot\vec{\delta r}[t]\times\vec{F}_1[t] + \{0,0,1\}\cdot\left(-\vec{\delta r}[t]\right)\times\vec{F}_2[t] \\ \dot{\vec{\phi}}[0] = 1^\circ \\ \vec{\phi}[0] = 133^\circ \end{cases}$$

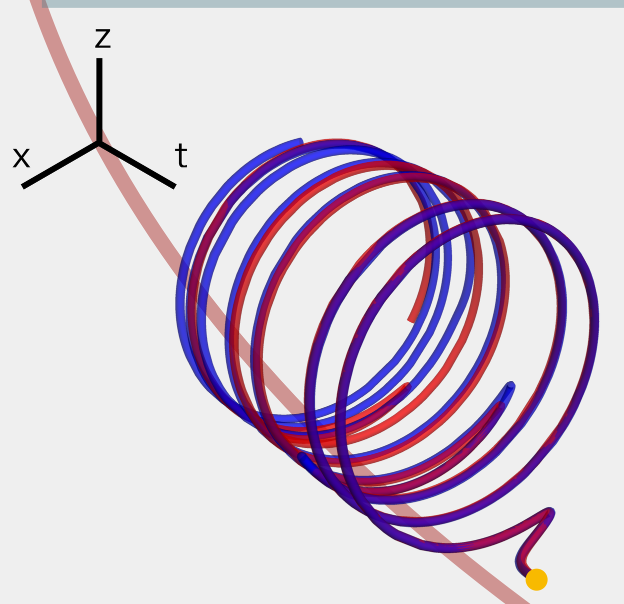
Polar Coordinate

$$\vec{r}[t] = \{x, y, z\} \to r[t]\{\cos\theta,\ \sin\theta,\ 0\}$$

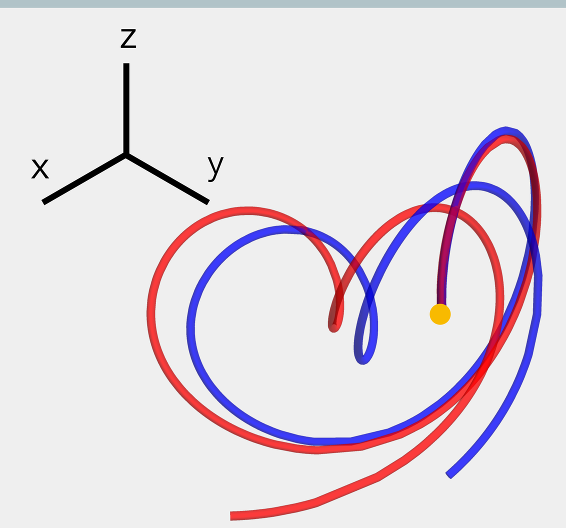
$$\begin{cases} \ddot{r}[t] - r[t]\dot{\theta}[t]^2 = \dfrac{-GM}{r[t]^2} \\ r[t]\ddot{\theta}[t] + 2\dot{r}[t]\dot{\theta}[t] = 0 \\ \dot{r}[0] = 0 \\ \dot{\theta}[0] = \dfrac{v_{\theta 0}}{r0} \\ r[0] = r0 \\ \theta[0] = 0 \end{cases}$$



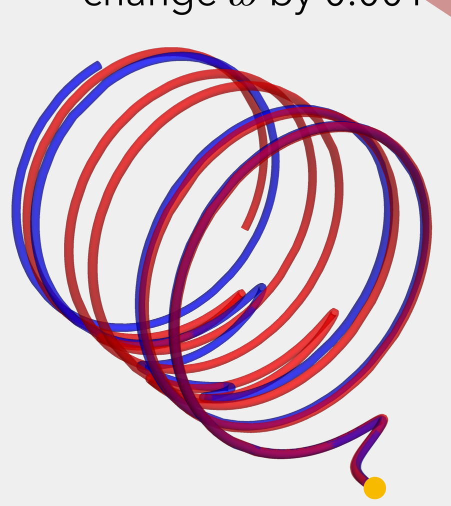
Chaos



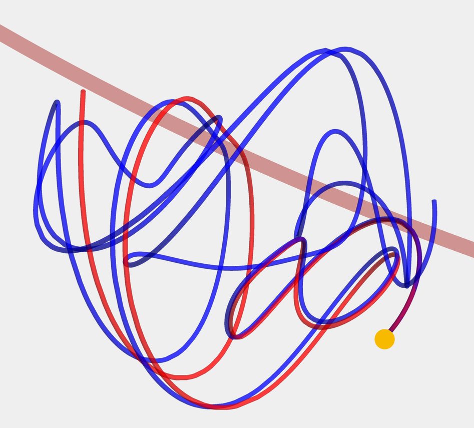
change ω by 0.001



change θ' by 0.001



change e by 0.01



change ϕ' by 0.001

3D Single Dumbbell

$$\vec{\delta r} = \delta r\{\sin\theta\cos\phi,\ \sin\theta\sin\phi,\ \cos\theta\}$$

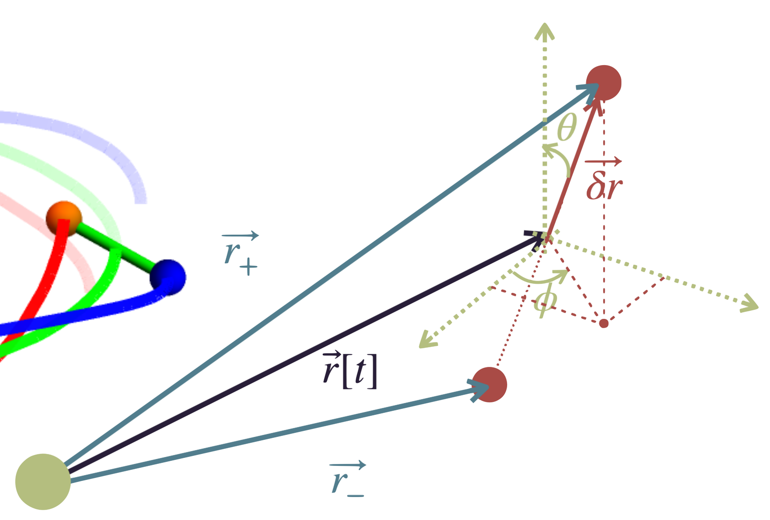
$$KE = \frac{1}{2}m\,\dot{\vec{r}}_+\cdot\dot{\vec{r}}_+ + \frac{1}{2}m\,\dot{\vec{r}}_-\cdot\dot{\vec{r}}_-$$

$$PE = -\frac{GMm}{r_+} - \frac{GMm}{r_-}$$

$$L = KE - PE$$

$$\frac{d}{dt}\frac{\partial L}{\partial\dot{\theta}} = \frac{\partial L}{\partial\theta},$$

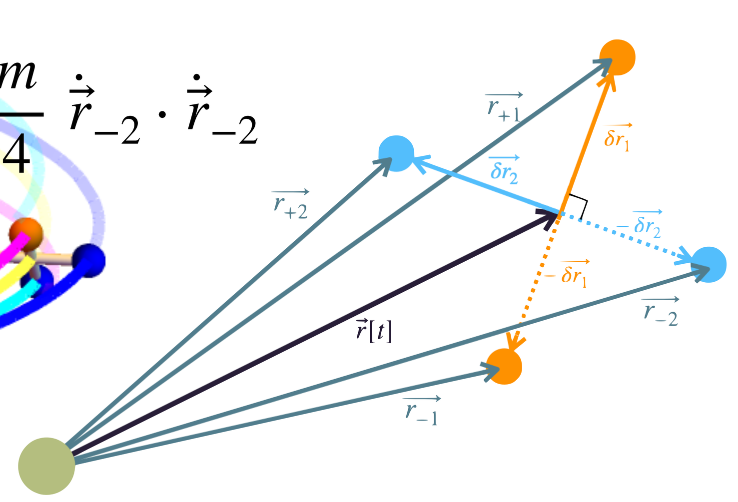
$$\frac{d}{dt}\frac{\partial L}{\partial\dot{\phi}} = \frac{\partial L}{\partial\phi}.$$



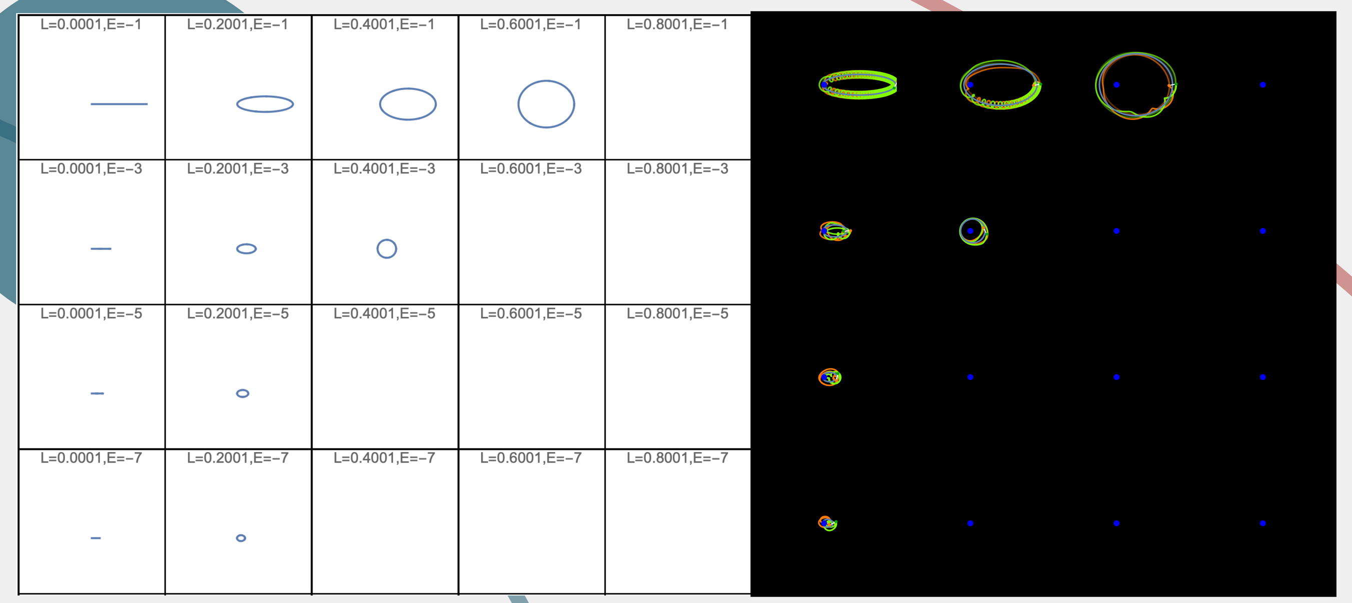
3D Crossed Dumbbell

$$KE = \frac{1}{2}\frac{m}{4}\,\dot{\vec{r}}_{+1}\cdot\dot{\vec{r}}_{+1} + \frac{1}{2}\frac{m}{4}\,\dot{\vec{r}}_{-1}\cdot\dot{\vec{r}}_{-1} + \frac{1}{2}\frac{m}{4}\,\dot{\vec{r}}_{+2}\cdot\dot{\vec{r}}_{+2} + \frac{1}{2}\frac{m}{4}\,\dot{\vec{r}}_{-2}\cdot\dot{\vec{r}}_{-2}$$

$$PE = -\frac{GMm/4}{r_{+1}} - \frac{GMm/4}{r_{-1}} - \frac{GMm/4}{r_{+2}} - \frac{GMm/4}{r_{-2}}$$



Varying angular momentum and energy



Varying eccentricity and larger radius