Angular frequency of rotating spiral waves in a chemical reaction-diffusion system

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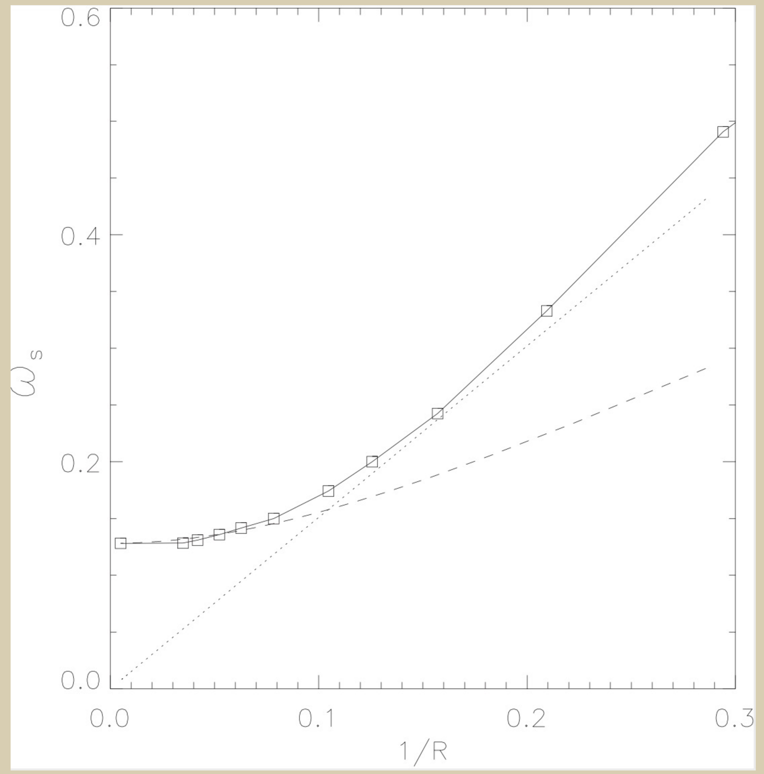
Abstract

Rotating spiral waves have been observed in various excitable physical, chemical, and biological reaction-diffusion systems. Most of the theoretical and experimental studies of two-dimensional excitable systems were done in planar geometries. However, in nature, many excitation waves occur on curved surfaces, e.g., the heart, the visual cortex, or the retina. In the framework of kinematic approach, it has been shown that the spiral wave's angular velocity depends on the curvature of the system itself. The chemical Belousov-Zhabotinsky reaction can be used as a model system to investigate the propagation dynamics of these spirals. We use quasi-two-dimensional hemispherical shells with various curvatures to determine experimentally the curvature dependence of the spiral's rotation frequency.

Theory

Research focus: Prove that the rotational frequency of a spiral wave increases as the system's curvature increases, using a liquid BZ solution.



[Zykov, 1996]

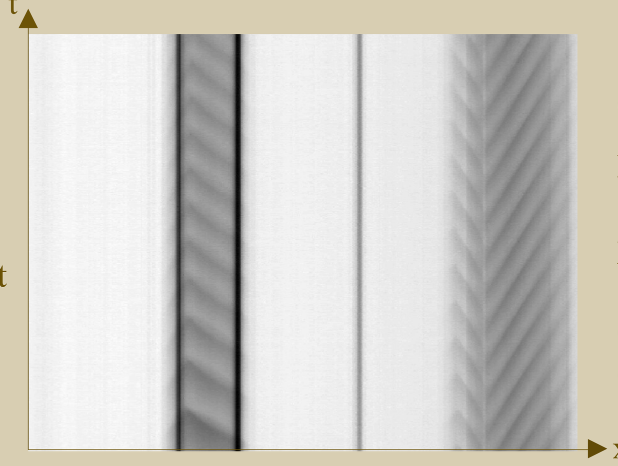
Small radius: $\omega_s = f_s(p) V_0 \frac{1}{R}$

Large radius: $\omega_s = \omega_0^2 + (1-p)^2 V_0^2 \frac{1}{R^2}$

Dimensionless parameter:
$p = D_u K_{crit} / V_0$

Results

Time-space plot for a spiral on a spherical cut (45 mm x 562 s).



Time-space plot for a spiral on a planar surface (44 mm x 660 s).

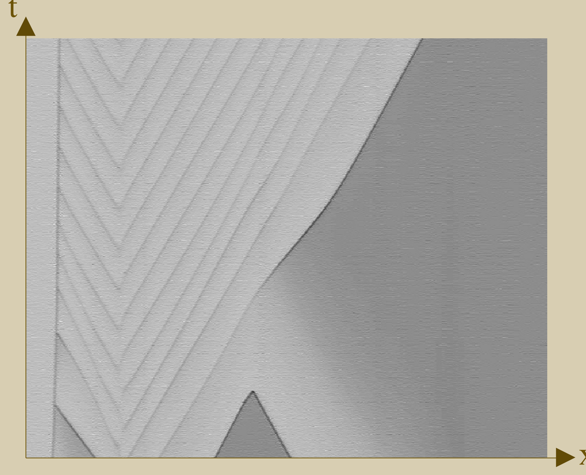
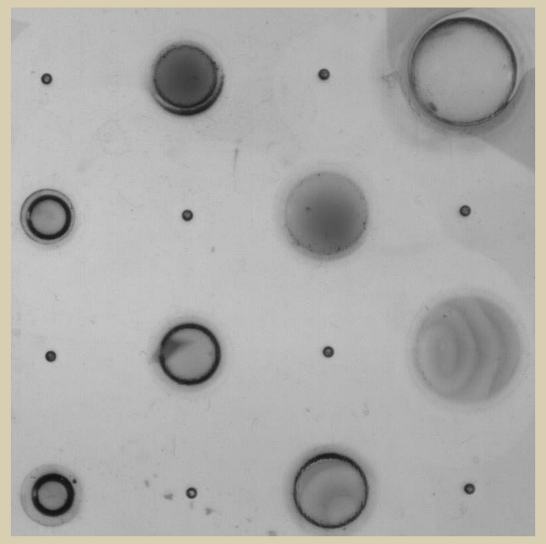
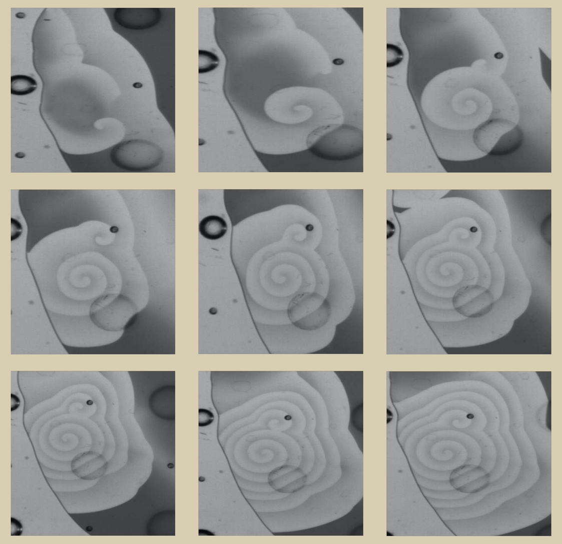


Image of a spiral on hollow spherical cuts of radius 12 mm and h= 2 mm, producing a curvature of $\frac{1}{12}$ mm^{-1} and a Gaussian curvature of $\frac{1}{144}$ mm^{-2}.



Video montage of a spiral on a planar surface, with $\Delta t =$ 50 s, corresponding to the time-space plot above.



Theory

- Spiral waves in chemical systems have been studied extensively on planar surfaces.
- Waves analogous to the electrical impulse waves traveling on the heart.
- These waves on curved surfaces have been investigated mainly theoretically.
- Spiral wave dynamics can be described using a simple two-variable reaction-diffusion model:

$$\frac{\partial u}{\partial t} = F(u,v) + D_u \Delta u,$$
$$\frac{\partial v}{\partial t} = \epsilon G(u,v) + D_v \Delta v.$$

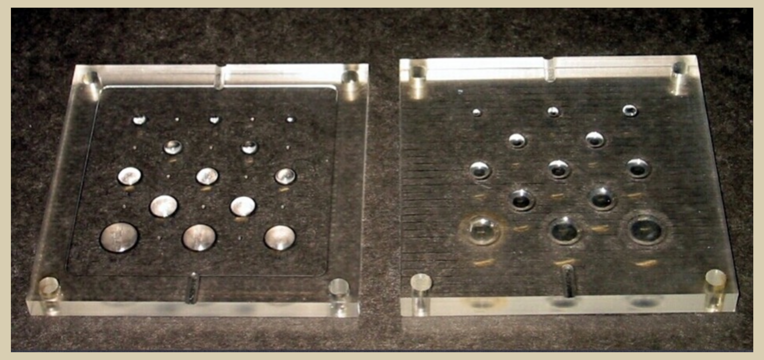
u = activator and v = inhibitor

Recipe & Molds

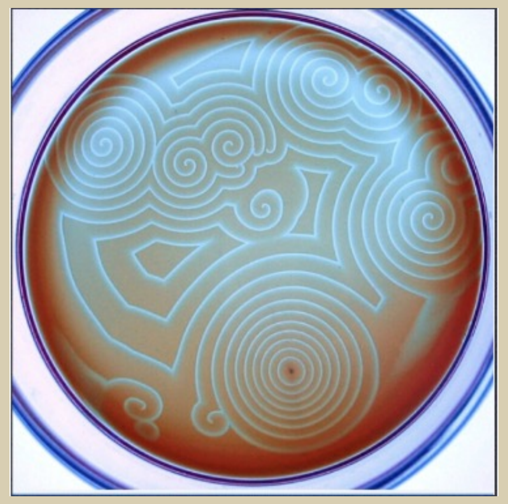
Bromate + Acid + Organic Substrate + Catalyst

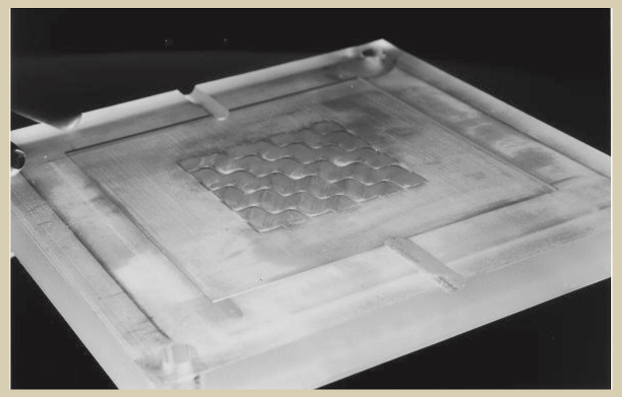
Malonic Acid →1,4-CHD

Gel mold→ Liquid solution



Spherical cuts, (75x75) mm^2

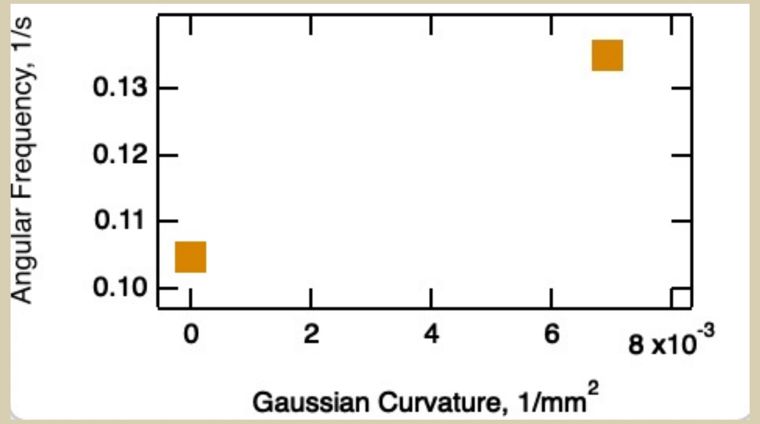




$f(x,y) = A \sin(\frac{2\pi}{\lambda} x) * \sin(\frac{2\pi}{\lambda} y)$ mold,
A = 1 mm & λ = 9 mm

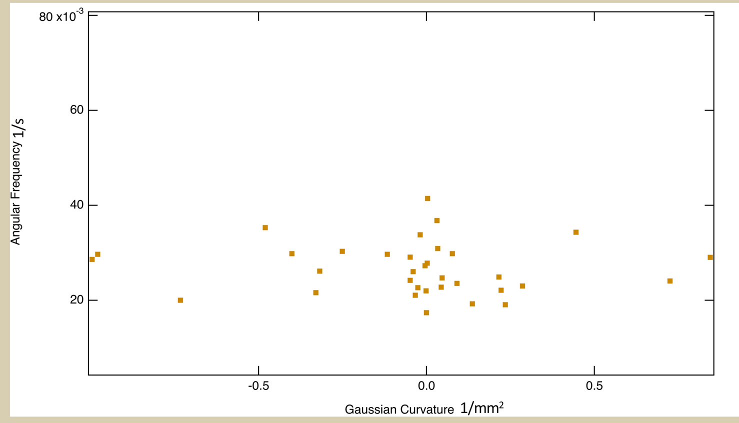
Conclusions

We start to produce results that match with theoretical and other experimental results and will continue to investigate smaller radii spherical cuts.



As angular frequency increases, Gaussian curvature also increases and agrees with Manz 2002.

Different results stem from the $\sin(x)\sin(y)$ mold, where we see that angular frequency does not change while Gaussian curvature increases.



As angular Gaussian curvature increases, angular frequency seems to stay around the same value.

References

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Manz2002: Niklas Manz. Untersuchungen chemischer Wellen in der Belousov-Zhabotinsky Reaktion: räumlich modulierte Systeme und anomale Dispersion (investigation of chemical waves in the Belousov-Zhabotinsky reaction: spatially modulated systems and anomalous dispersion). PhD Thesis, Otto-von-Guericke Universität Magdeburg, Magdeburg, 2002.

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