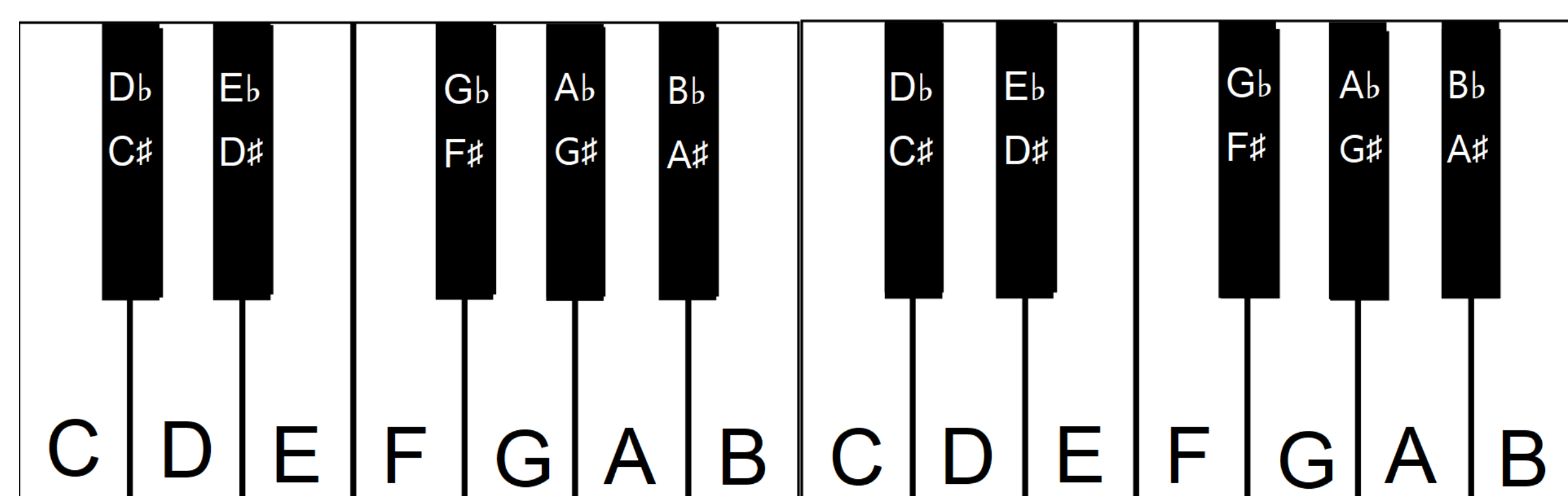


Music Theory

- **12 tones of equal tempered music:** Ab, A, Bb, B, C, Db, D, Eb, E, F, Gb, G
- A **chord** is formed of three or more tones. For example, C, Eb, and G form a minor chord.



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History

- *Tonnetz* translated from German is literally “tone network.”
- The Tonnetz was discovered by Leonhard Euler in 1739. In Euler’s Tonnetz to the right, the “H” represents “B” while “B” represents “Bb.” This comes from German Renaissance music theory notation.
- Carl Ernst Naumann (1832–1910) constructed the more traditionally recognized format of the Tonnetz.

The Tonnetz shows relationships between chords.

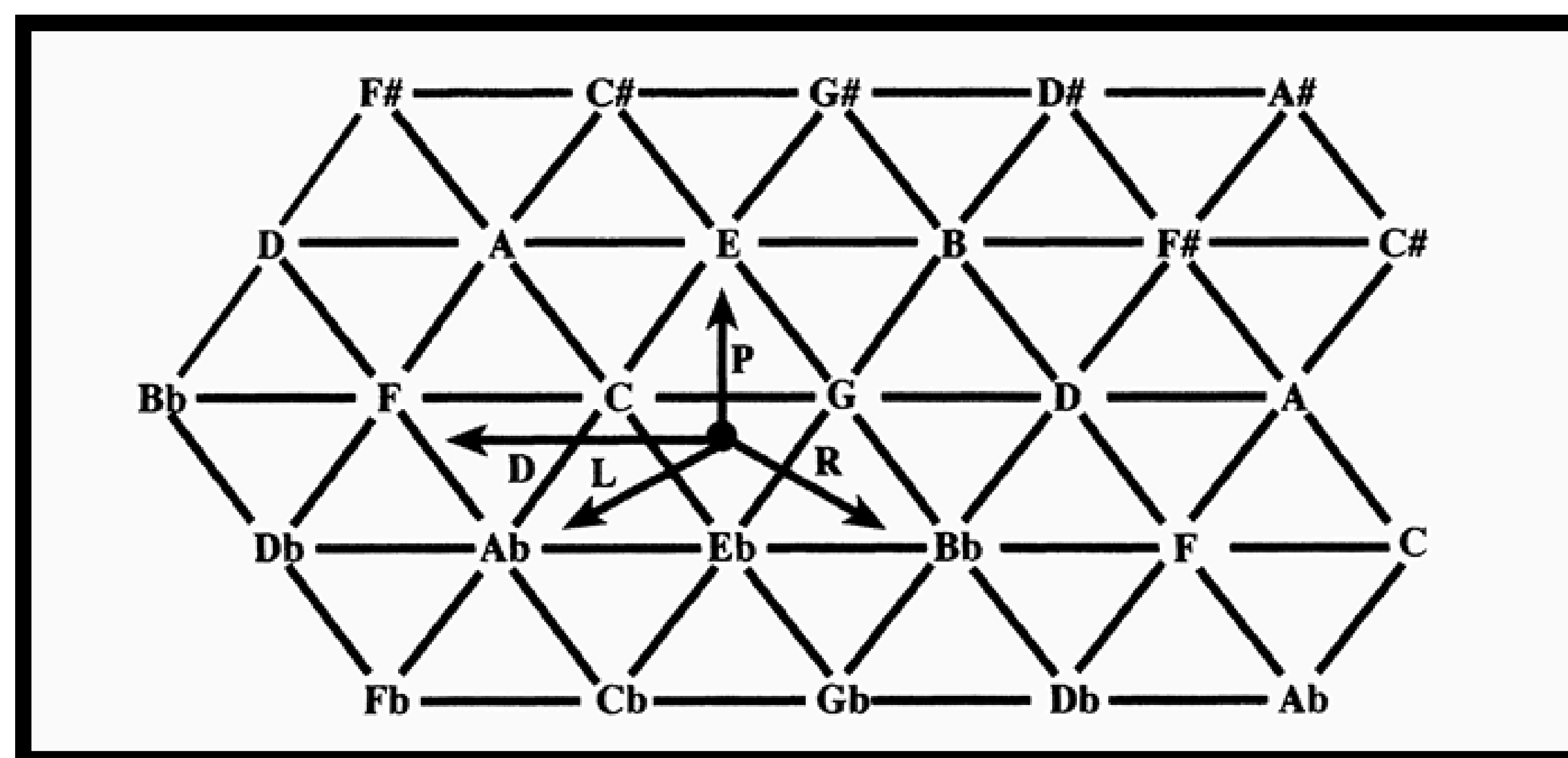
Hugo Riemann (1849–1919) developed two classes of transformations of triads called a *Schritt* (“step”) and a *Wechsel* (“swap”). David Lewin, in 1982, extended the basis of Riemann’s two class transformations into the

Neo-Riemannian Transformations:

- **Relative Major/minor (R):** ex. (C, Eb, G) – (G, Eb, Bb)
- **Parallel Major/minor (P):** ex. (C, Eb, G) – (C, E, G)
- **Leading-Tone Exchange (L):** ex. (C, Eb, G) – (Ab, C, Eb)
- **Dominant (D)** is redundant as it is a combination of L and R transformations.

A Link Between Mathematics and Music: The Tonnetz

The Note-Based Tonnetz

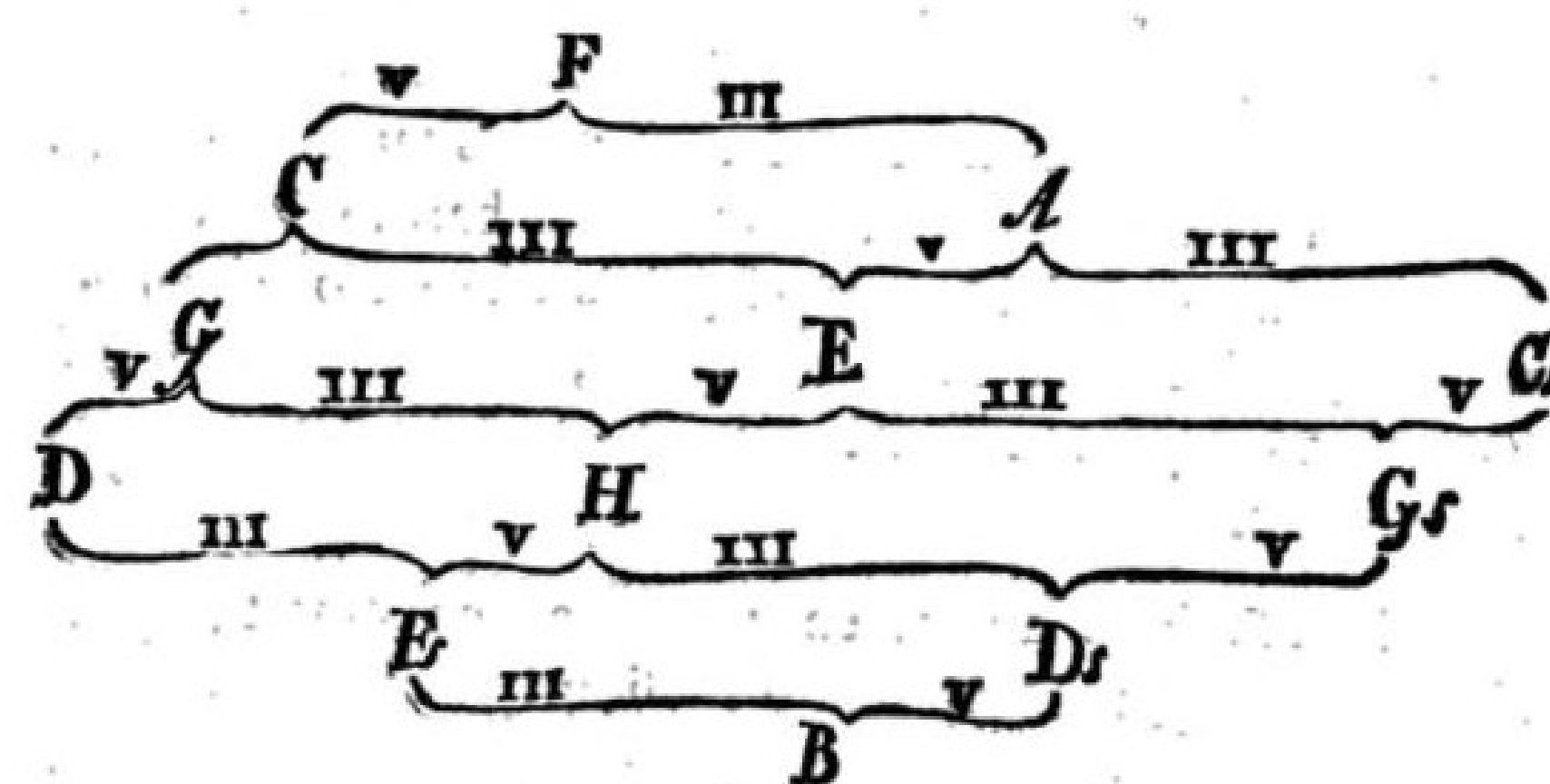


Richard Cohn. “Introduction to Neo-Riemannian Theory: A Survey and a Historical Perspective.” *Journal of Music Theory*, 1998, Vol. 42, No. 2, pp. 167–180.

The space of the note-based Tonnetz

Unordered pitch equivalence class space

- **Pitch equivalence class** disregards the octave of a tone. The 12 tones are the 12 pitch equivalence classes.
- **Unordered pitch** disregards the order of notes in a chord.
 - We are only interested in the *acoustic* qualities of chords.
 - *Voice-leading* qualities must be represented in ordered pitch class space or “chord space.”
- The **space** of the Tonnetz is a *quotient space*.



Leonhard Euler, et al. “Tentamen novae theoriae musicae ex certissimis harmoniae principii dilucide expositae.” *Ex typographia Academiae Scientiarum*, 1739.

Mathematics

Formal Definitions:

Let X be a set. An **equivalence relation** (denoted \sim) on X has the following properties:

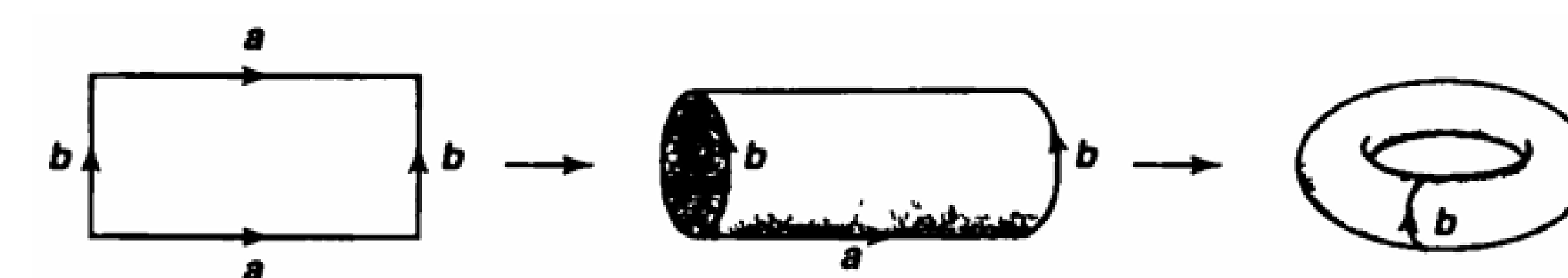
- (Reflexive) Let $a \in X$. Then, $a \sim a$.
- (Symmetric) Let $a, b \in X$ such that $a \sim b$. Then, $b \sim a$.
- (Transitive) Let $a, b, c \in X$. If $a \sim b$ and $b \sim c$, then $a \sim c$.

An **equivalence class** $[x]$, of an element $x \in X$ is the set $[x] = \{y \in X : x \sim y\}$ for a given equivalence relation.

Example:

The rational numbers form equivalence classes under the relation $a/b \sim c/d$ when $ad=bc$, so $[1/2] = \{\dots, -2/4, -1/2, 1/2, 2/4, \dots\}$.

A **quotient space of X** is the set of an equivalence class (along with some additional structure) and is denoted X/\sim .



Quotient space of a rectangle can be a torus. Image from James Munkres’ *Topology*, pp. 136.

Selected References

- [1] *The Oxford handbook of Neo-Riemannian music theories*. Oxford handbooks series. New York: Oxford University Press, 2011.
- [2] Dmitri Tymoczko. “The Generalized Tonnetz.” *Journal of Music Theory*, 56: 1, 2012, pp. 1–52.
- [3] James Raymond Munkres. *Topology*. Upper Saddle River, NJ: Prentice Hall, 2000.