Tiling the Plane, Breaking the Rules: Aperiodicity and Infinity in Tiling Theory

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Overview

- Aperiodicity
- Wang Tiles
- Penrose Tiles
- The Taylor-Socolar Tile
- The Einstein Tile
- Infinity and Applications

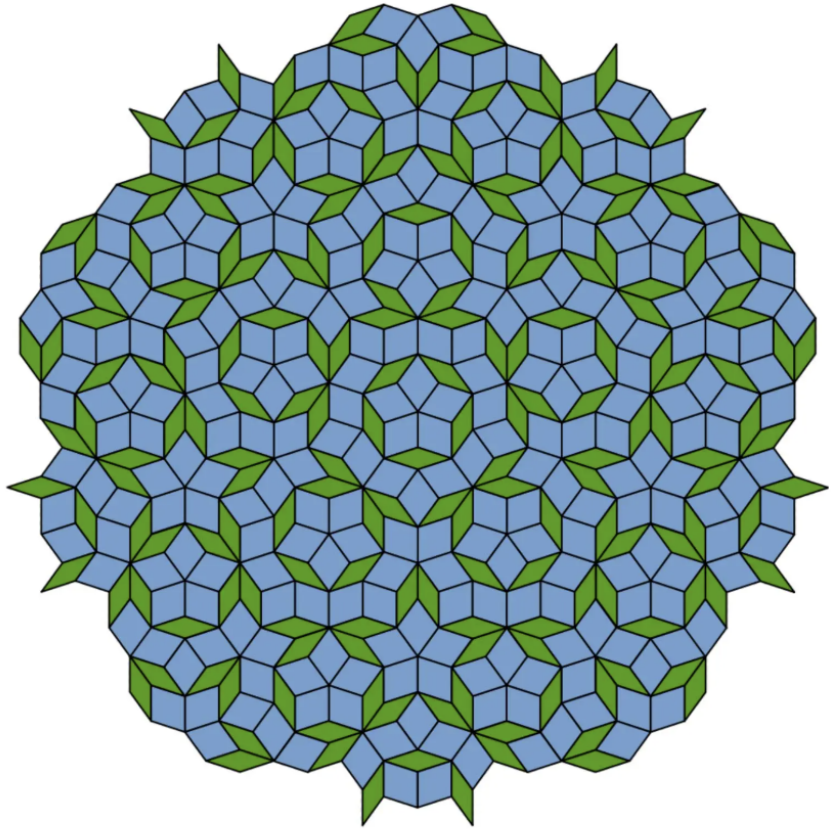

Figure 1: Penrose thick and thin rhombi tiling [Klarreich, 2023]

Aperiodicity

Aperiodicity

- Aperiodic tilings force non-repetition across an infinite region
- The pattern never settles into a repeating unit
- Periodic tiling: a translation repeats a fundamental region, so the tiling is built from a finite repeating unit cell
- Aperiodic tiling: irrational relative density means that the tiling cannot come from any repeating unit and no translation can reproduce the tiling

Wang Tiles

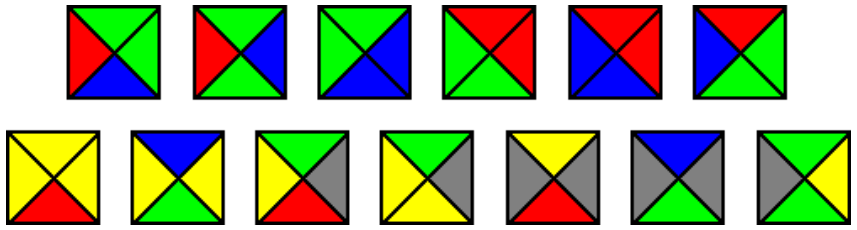

Figure 2: A set of Wang Tiles [Wikipedia contributors, 2025]

- Colored unit squares that are broken into four triangles
- Entscheidungsproblem
- Basis for the Domino Problem: given a finite set of Wang tiles, can it tile the plane?
- Wang conjectured that if a finite set of Wang tiles can tile the infinite plane, then the tiling must be periodic
- Student Robert Berger proves this conjecture wrong with an aperiodic tiling of 20,426 Wang tiles

Penrose Rhombi

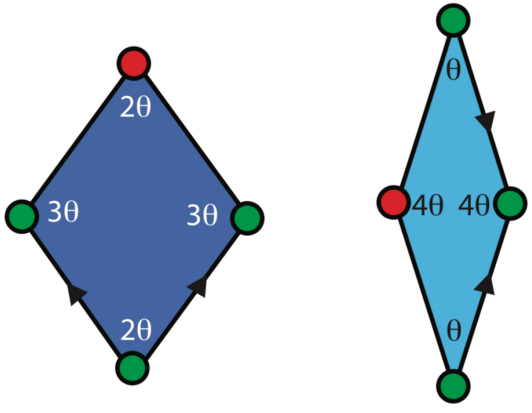

Figure 3: Penrose Thick and Thin Rhombi [Adams, 2023].

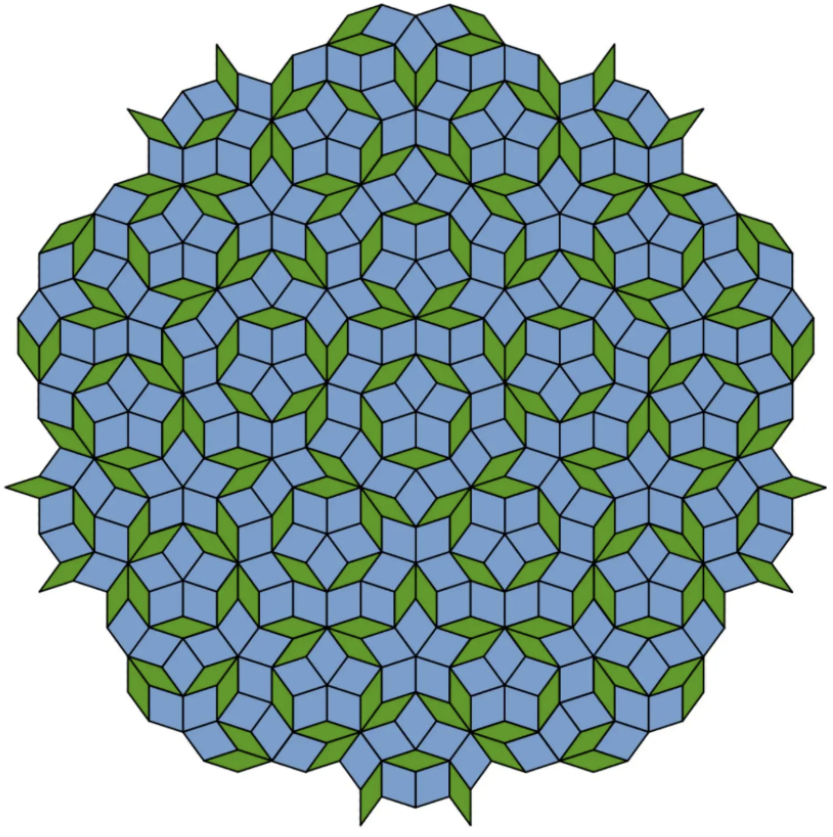

Figure 4: Penrose thick and thin rhombi tiling [Klarreich, 2023]

Penrose Kite and Dart

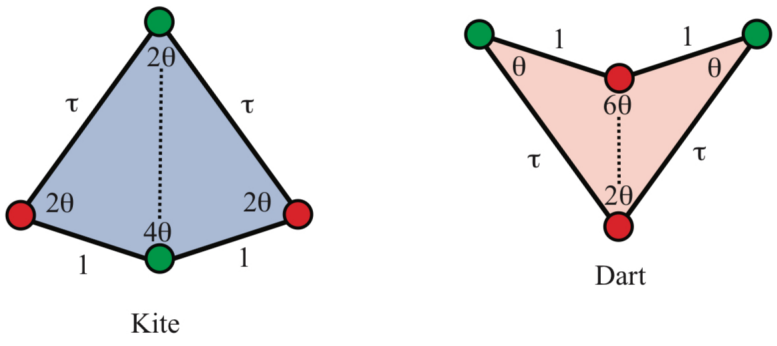

Figure 5: A Kite and Dart, where $\tau = \Phi =$ the golden ratio [Adams, 2023].

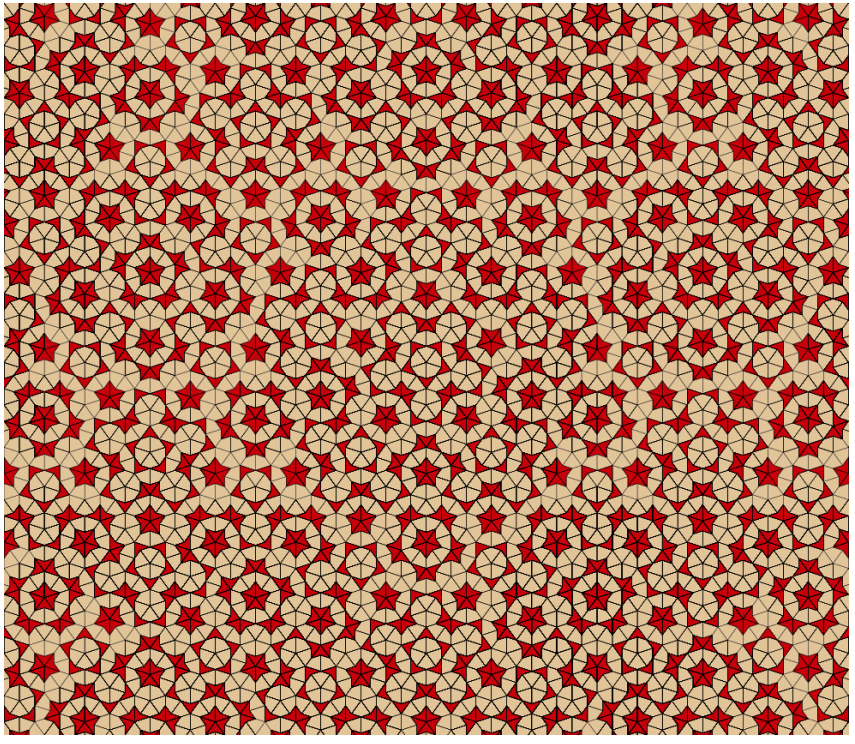

Figure 6: Penrose kite and dart tiling

Taylor-Socolar Tile

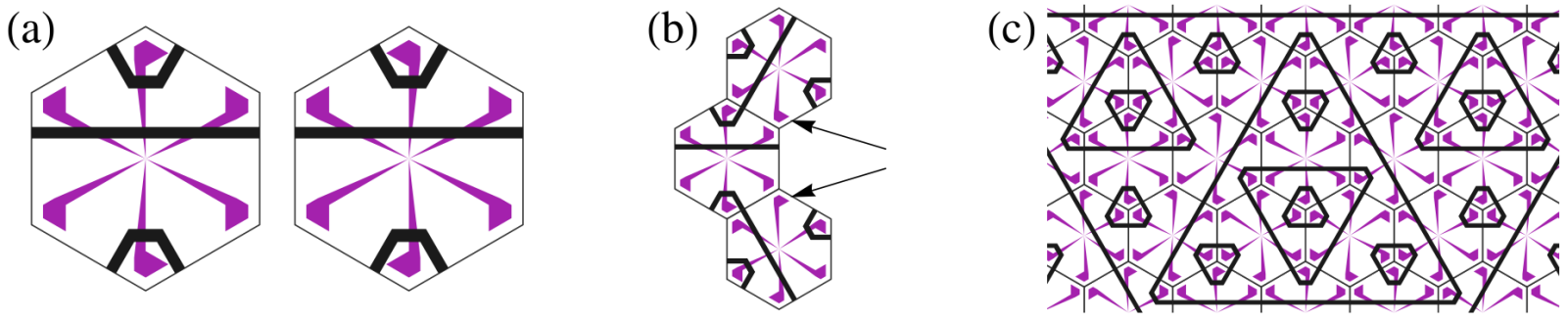

Figure 7: A decorated hexagonal aperiodic monotile with matching rules. [Socolar and Taylor, 2011]

The ‘Einstein’ Tile

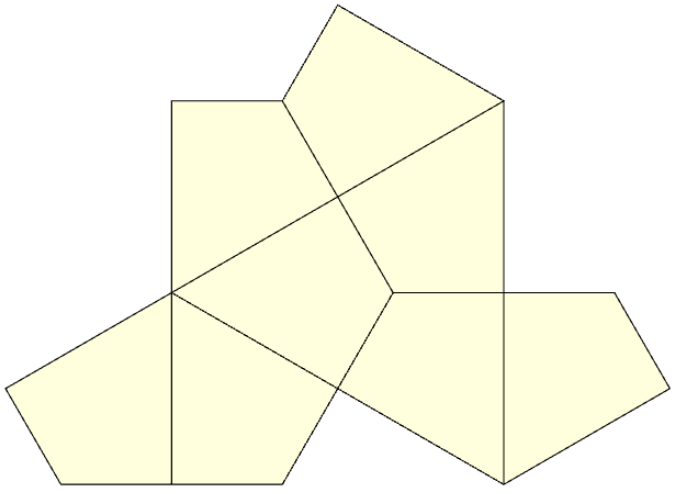

Figure 8: The einsten tile or the hat polykite [Weisstein, 2024]

- Introduced in 2023 by David Smith, Joseph Samuel Myers, Craig S. Kaplan, and Chaim Goodman-Strauss
- German phrase ein Stein, meaning “one stone”
- Works without edge-coloring or matching constraints
- Forms hierarchical structure (self-similar patterns emerge)
- Enforces non-periodicity through geometry alone

Infinity

Infinity in Aperiodic Tilings

- Aperiodic tilings cover the plane infinitely in all directions without repeating patterns
- Local rules can generate non-repeating structures at all scales
- Inflation/deflation creates self-similar structures across infinite scales and regions
- Infinitely many distinct patches appear and no finite region can capture the whole tiling
- Some tiles belong to an infinite family of aperiodic monotiles

A Family of Aperiodic Monotiles

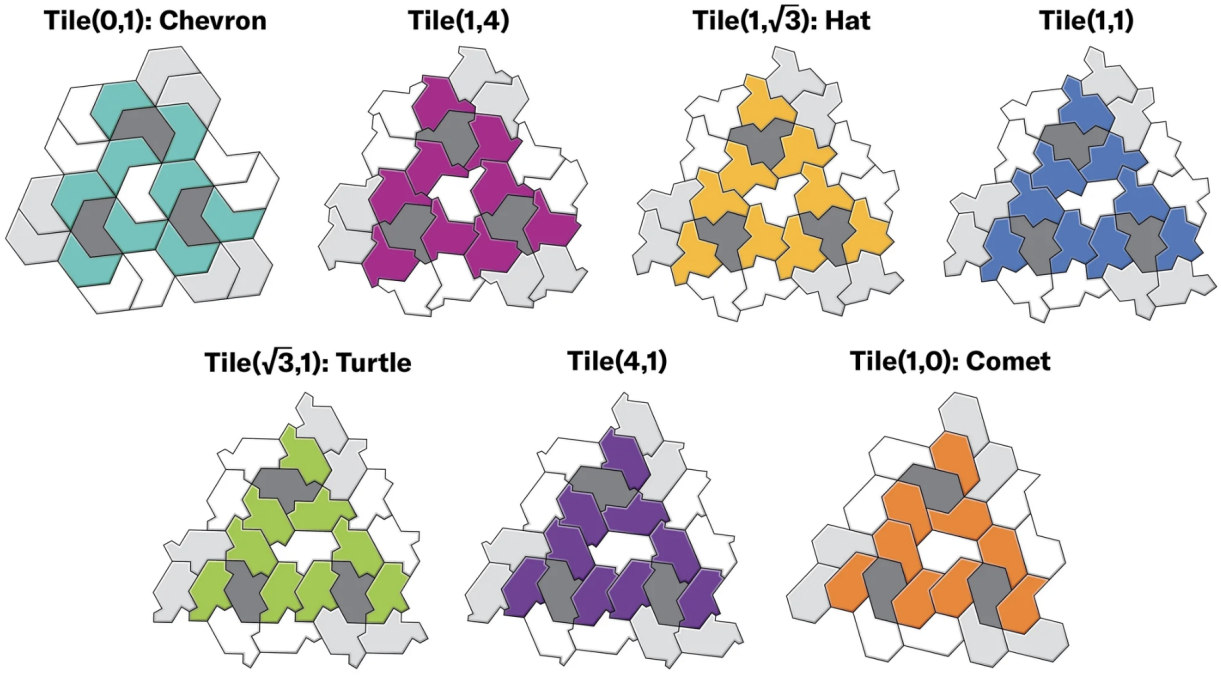

Figure 9: A Family of Aperiodic Monotiles [Kaplan, 2024]

Applications In Other Fields

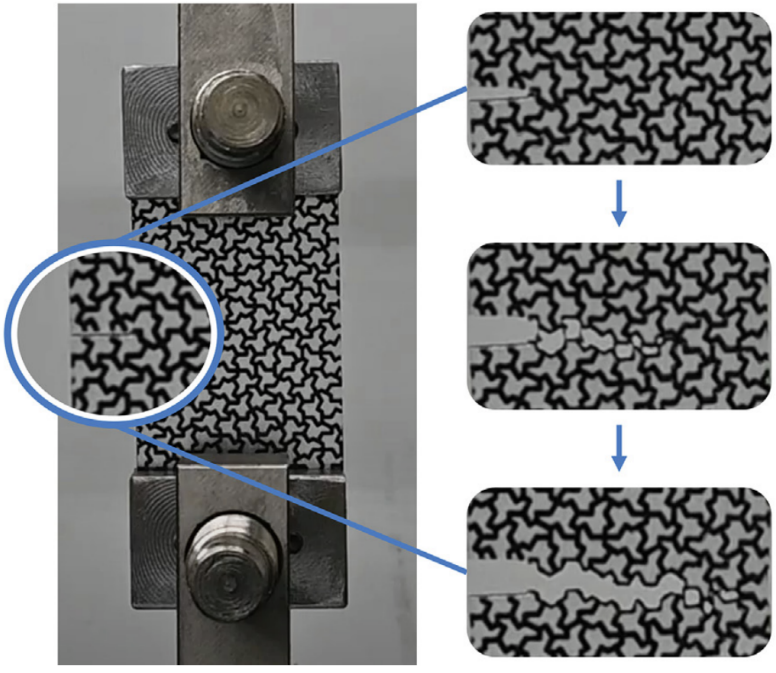

Figure 10: An example of the crack pattern that arises when intense pressure is put on materials composed of aperiodic monotiles [Jung et al., 2024]

Engineering and Aperiodic Monotiles

- High-performance composite materials
- Enhanced stiffness, strength, and toughness
- Exhibits mechanical isotrophy
- Used in airplanes, automotive crash structures, and orthopedic joint replacements

Thank You! Questions?

References I


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