The Memory of Light: Investigating a Nonlinear Phase Accumulation in the Polarization Transformations of Light

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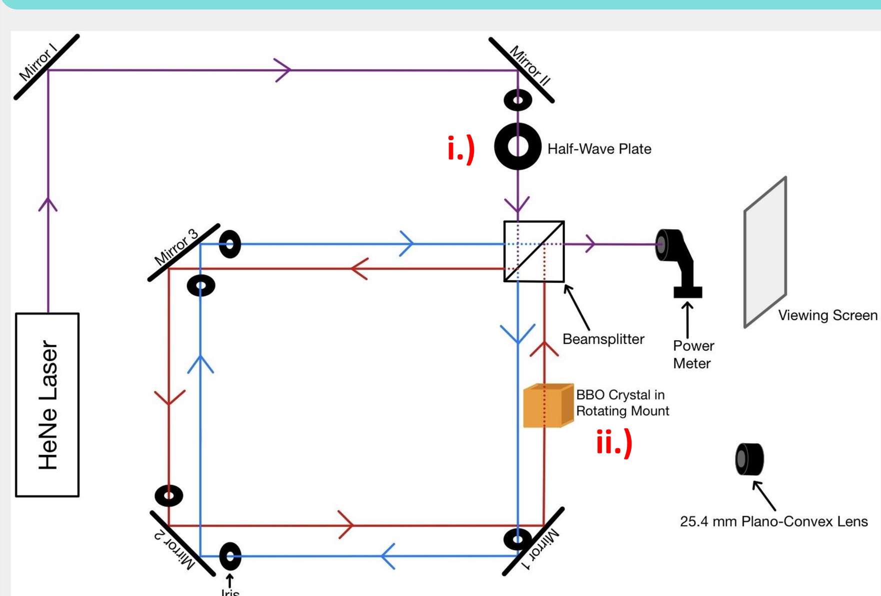
Abstract

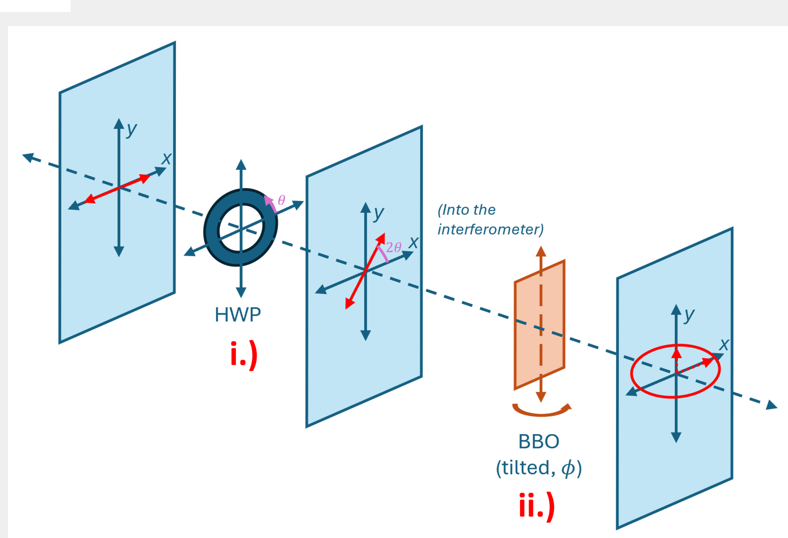
This work investigates a nonlinear, geometric phase accumulation resulting from polarization transformations of coherent light. A horizontally linearly polarized beam first passed through a half-wave plate with a variable rotation angle, controlling the angle of the linearly polarized light entering a dual-path interferometer. Within the interferometer, the beam was split into a reference path, which retained the polarization set by the half-wave plate, and a path passing through a barium borate (BBO) crystal, which was tilted incrementally. Tilting the crystal caused the polarization of this beam to evolve continuously through a range of ellipticities. Recombining the two beams produced interference patterns with varying visibility and extrema amplitudes. Experimental data revealed characteristic beating patterns arising from the crystal's birefringent properties. Comparison of extrema positions between runs with distinct input polarization states reveals patterns suggestive of a rapid phase shift approaching π. While a precise quantitative measurement of the accumulated phase was not determined, these results provide clear qualitative experimental evidence of geometric phase accumulation, in agreement with predictions from a heuristic model and the Poincaré sphere representation, demonstrating geometric phase induced by birefringent-crystal-based polarization transformations.

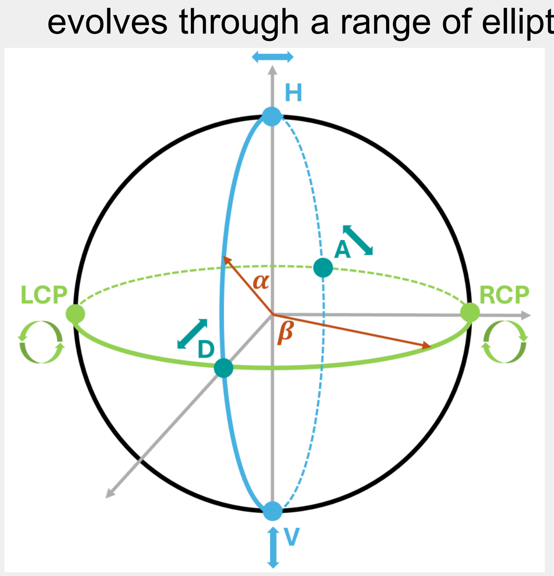
Theory



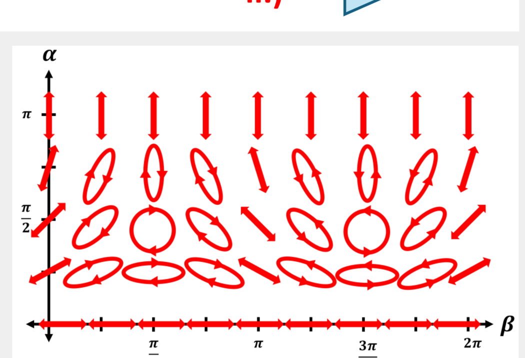
Horizontally polarized light of wavelength $\lambda = 632.8$ nm is incident on a half-wave plate (HWP) which controls the polarization state of the light entering the interferometer. The **transmitted** beam (blue) maintains this initial polarization and is therefore the reference beam. The **reflected** beam (red) passes through a birefringent BBO crystal which causes its polarization state to continuously evolve as the crystal is rotated through a range of tilt angles. The beams recombine and the resulting **interference** is analyzed using a power meter.

- Turning the HWP through an angle θ rotates the orientation of the linearly polarized light entering the interferometer by 2θ.
- The BBO crystal introduces a **phase delay** between the x and y components of the polarization, changing it from **linear to elliptical**. This phase delay varies as the tilt angle ϕ of the crystal is altered. Thus, as the crystal is rotated through a range of angles, the polarization of the light passing through it evolves through a range of ellipticities.





- Every possible polarization state is mapped to the surface of the Poincaré sphere [1].
- HWP rotates the state along a line of longitude (alters $\alpha = 4\theta$).
- Tilting the BBO crystal traces a path of latitude (alters β).
  - If this path is closed with a geodesic, **the geometric phase equals half the enclosed area** [1].

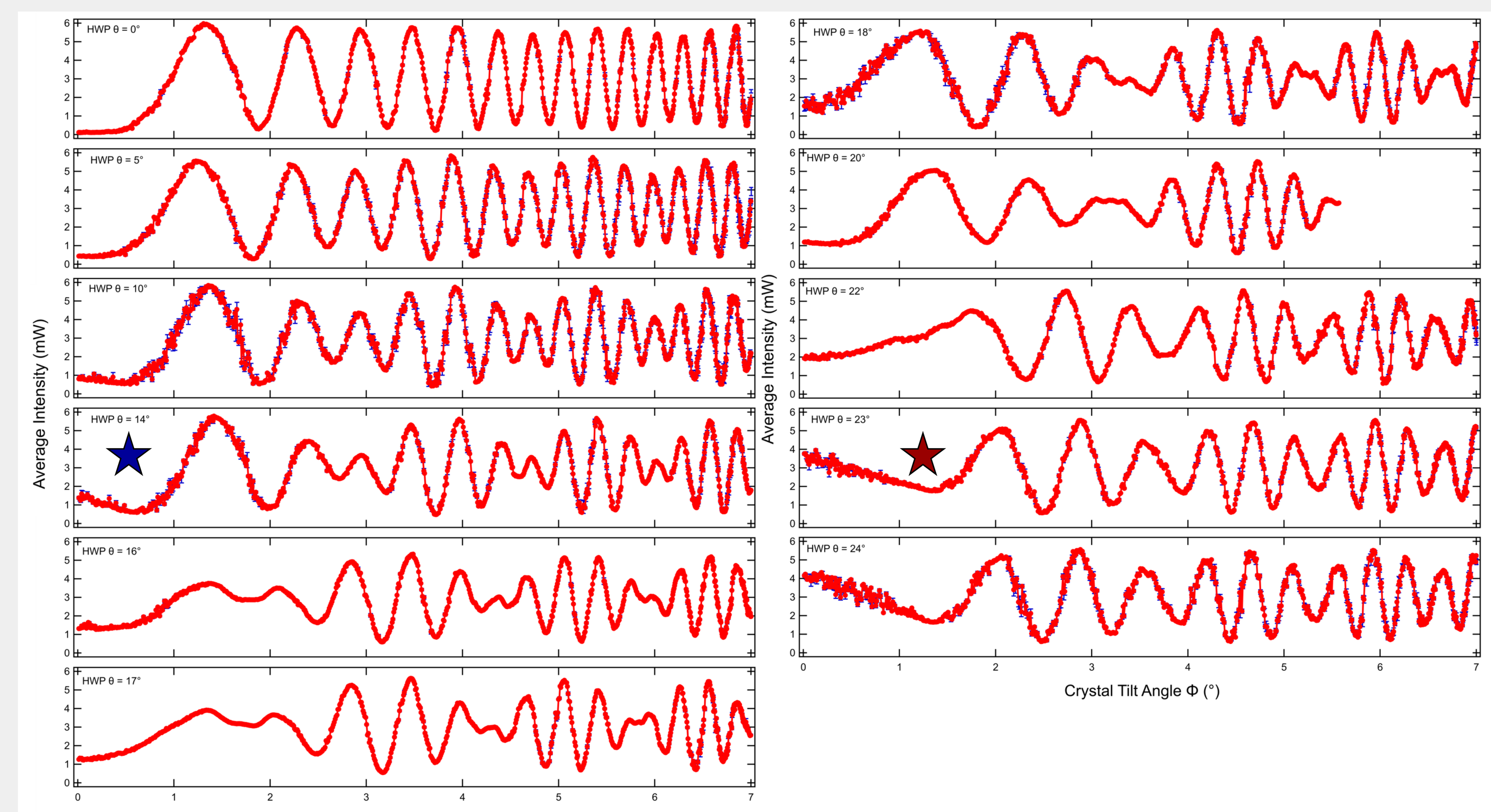


A 2-D representation of the states mapped to the Poincaré sphere.

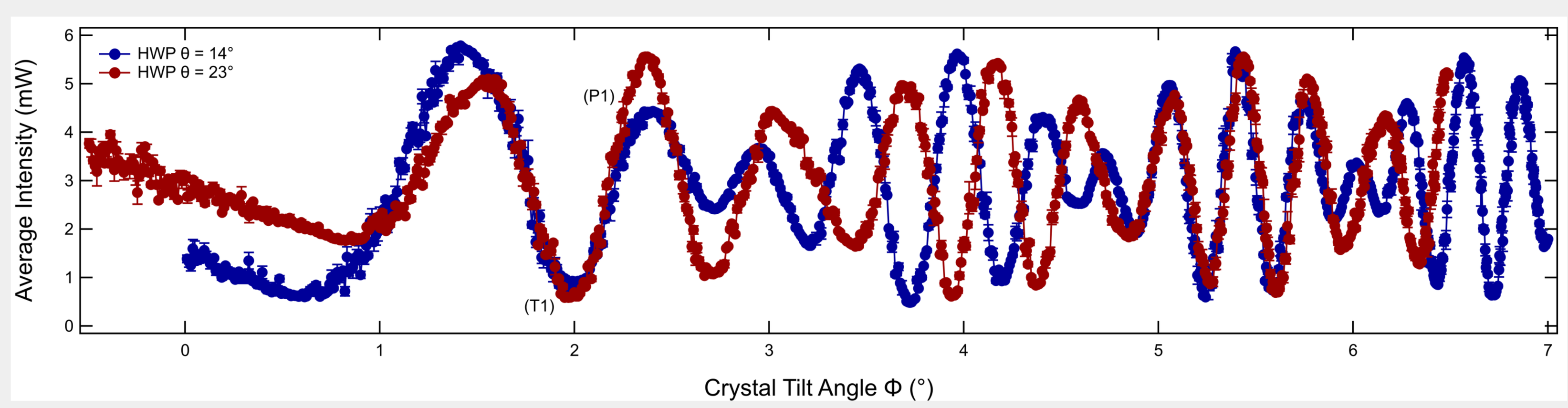
The interference effects from the BBO crystal on the polarization of light, and thus the accumulation of geometric phase, can be modeled with the following **intensity** equation [1].

$$I_{out} = \frac{1}{2}|E_0|^2\left[1 - \sqrt{\cos^2\left(\frac{(k_x - k_y)z}{2}\right) + \sin^2\left(\frac{(k_x - k_y)z}{2}\right)\cos^2(\alpha)} \times \cos\left(\frac{(k_x + k_y)z}{2} - \tan^{-1}\left(\frac{\sin\left(\frac{(k_x - k_y)z}{2}\right)\cos(\alpha)}{\cos\left(\frac{(k_x - k_y)z}{2}\right)}\right)\right)\right]$$

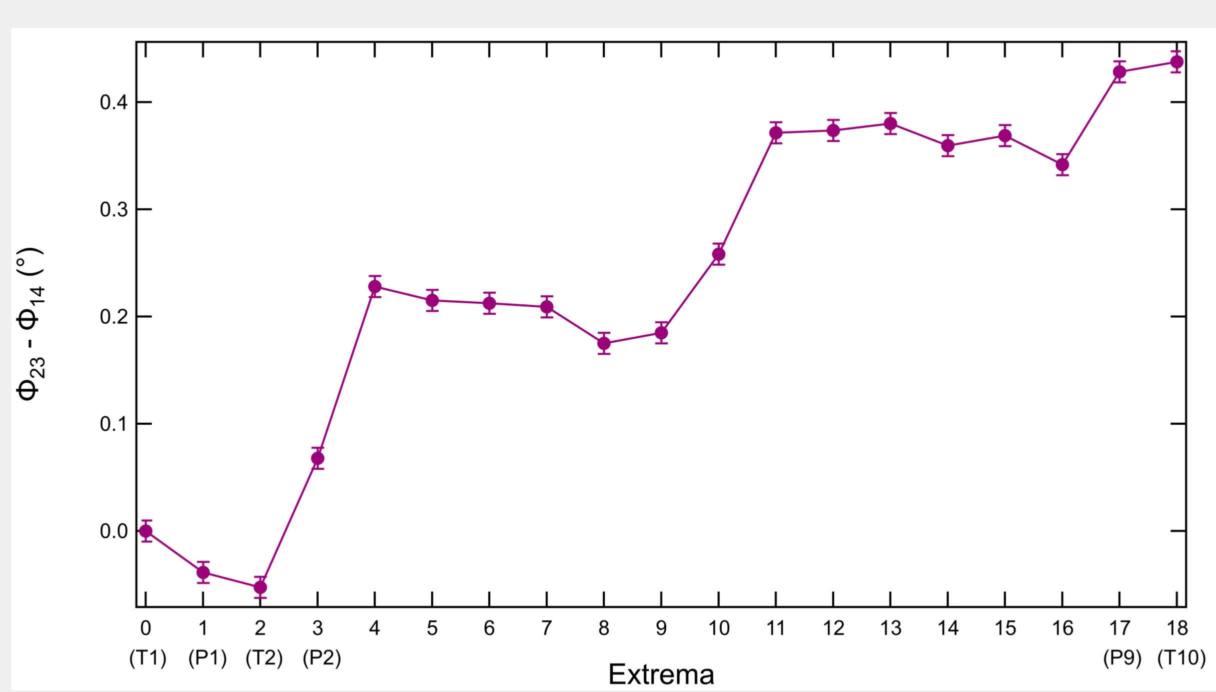
Experimental Results



Average intensity vs. crystal tilt angle ϕ for the BBO crystal with a horizontal optic axis orientation. The data correspond to HWP angles ranging from $\theta = 0°$ to $\theta = 24°$. The blue vertical error bars represent the standard deviation of the measured intensity values.



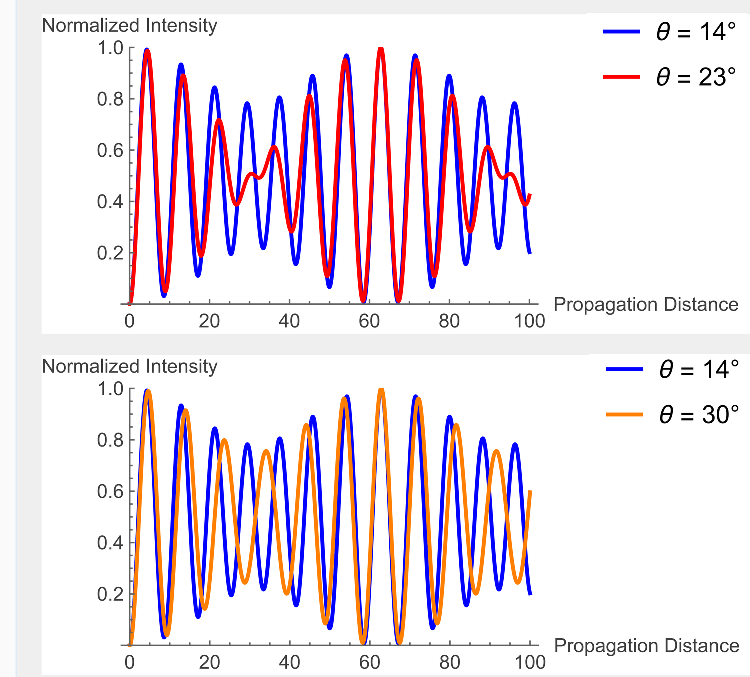
Comparing average intensity vs. crystal tilt angle ϕ with the BBO crystal at a horizontal optic axis orientation for HWP angles of $\theta = 14°$ (blue) and $\theta = 23°$ (red). The blue and red vertical error bars represent the standard deviation of the measured intensity values. These runs are aligned by their first defined troughs, labeled as (T1). A phase shift approaching π can be observed between the two runs as they approach $\phi = 4°$.



Difference in crystal tilt angle ϕ at the extrema between the $\theta = 23°$ and $\theta = 14°$ runs. This analysis begins at the first aligned trough, labeled as (T1). The purple vertical error bars are given by the uncertainty of $\phi_{23} - \phi_{14}$.

- Upward trend with a repeating pattern of peaks and valleys.
- Extrema in the $\theta = 23°$ run occur at larger tilt angles.
- Peaks and troughs exhibit a pattern of small decreases, followed by a large increase and then areas of constant difference, producing a repeating oscillatory structure.

Modeling



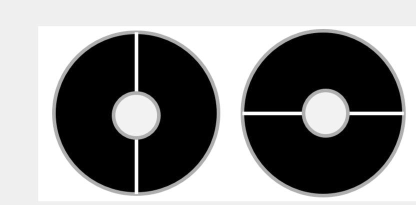
- Modeled normalized intensity as a function of propagation distance through the BBO crystal, each comparing runs at two different half-wave plate angles.
- The blue curves represent the half-wave plate angle $\theta = 14°$.
- On the top plot, the red curve corresponds to a half-wave plate angle of $\theta = 23°$.
- On the bottom plot, the orange curve corresponds to a half-wave plate angle of $\theta = 30°$.

Conclusions

- Qualitative agreement between experimental data, and predictions from a heuristic model and the Poincaré sphere representation.
- Identified characteristic beating patterns due to birefringence.
- Demonstrated geometric phase accumulation induced by birefringent-crystal-based polarization transformations.
- Comparisons between extrema positions of two different runs revealed a phase shift approaching π.

Future Work

- Investigate vertical crystal orientation.



Vertical vs. horizontal crystal optic axis orientation.

- Extend crystal tilt angle range.
- Improve consistency in low-angle measurements.
- Enhance automated rotation stage software.
- Develop a more predictive model that accounts for the complex properties of the BBO crystal [2].

Acknowledgments

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References

- [1] Jonathan D. Logan. Measuring a Nonlinear Phase Accumulation for Cyclic and Non-cyclic Adiabatic Transformations of the Polarization State of Light, 2022. [Undergraduate thesis from The College of Wooster, Department of Physics].
- [2] Al M. Cawley. Computationally Modeling Phase Accumulations within a Birefringent Crystal System, 2025. [Undergraduate thesis from The College of Wooster, Department of Physics].