

Calculated Risk: A Game-Theoretic and

Probabilistic Analysis of Texas Hold 'em Poker

Christian Kring

The College of Wooster

Department of Mathematics



Objectives and Approach

- Analyze what kinds of advantages can be gained through applying game theory and mathematics to the game of Texas Hold 'em Poker.
- Finding such advantages would allow players to play more consisted and with more logic, leading to much greater results.
- Advantages and finds should be simplified as much as possible to make them easy to interpret for any player.

Imperfect Information

- Using the logic of combinatorics, we analyzed what the conditions are for each specific hand rank to be possible for a two-card hand, based on the set of community cards, which allowed us to simplify the various hand rankings an opposing player could have, substantially decreasing the outcomes to be considered.
- Connected the poker concept bucketing to various set theory terms such as equivalence relations and partitioning, which made dealing with card combinations much more simple.



Nash and Bayes' Equilibrium

- Nash Equilibrium: Seeks to find the most optimal state of the game while emphasizing mixed strategy.
- Bayesian Equilibrium: Optimizes conditional probability through belief updating. $P(A | B) = \frac{P(A)P(B | A)}{P(B)}$
- Can use each of the concepts in different ways, but both look to make a player's action's optimal, and maximize their expected value.

Game One Application: Ivey vs Negreanu

- Created a decision tree that represented a player's net outcome based on the choices that he makes, and whether or not he has the winning hand.
- Applied Bayes' Rule to the probability of whether Negreanu's hand received greater value due to the addition of a new card, given the event that he raised.
- Analyzed the specific buckets in which Phil's hand would beat Daniel's if the game were to go to a showdown.



Preflop Considerations and Ranges

- Distinguished the mathematical possibilities of preflop poker when compared to later event within a round.
- Considered four possible strategies specific to a range chart and created an expected value function represent each.
- Found that each strategy distribution for each two-card hand will balance optimization with unpredictability, in order to make a player less exploitable.
- Used multiple functions to represent preflop values such as total pot size and each player's contributions.

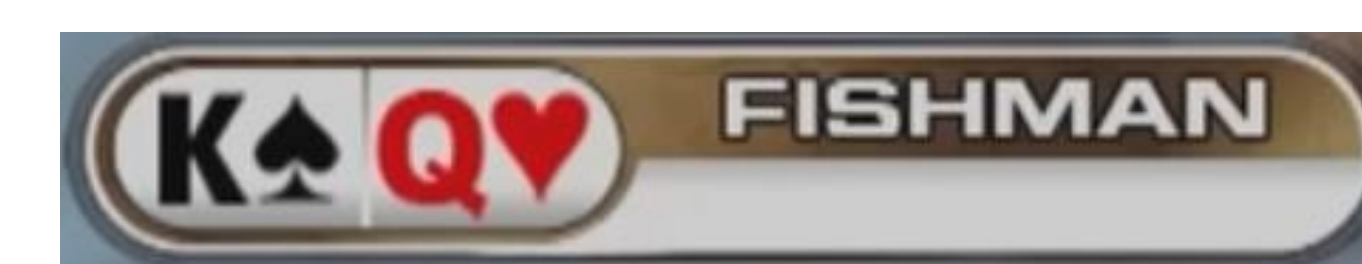
$$(E(v_h))(p_{vh}) + (E(b_h))(p_{bh}) + (E(l_h))(p_{lh}) + (E(f_h))(p_{fh}) \quad Y = \frac{3x}{2} + x \sum_{i=1}^n (b_i + d_i)$$

Signaling Theory

- The relationship between a sender and receivers in which beliefs may be updated according to perceived actions.
- Heavily varies based on each player's hand, as well as the actions being made.
- Further addressed the concept of Nash Equilibrium and mixed strategy, as well as a player's expected value.

Game Two Application: Fishman the Math Teacher

- Counteracts signaling theory and disguises the strength of his hand through a raise by another player (5 times the big blind).
- Considered the positives and negatives of four players staying in the pot.
- Used information states to represent the possible outcomes of the game based on the choices made by each player.



Conclusions

- Although it can be difficult to fully apply the concepts of game theory and mathematics, there are still many ways poker players can use the two to get an advantage while playing.
- Simplifying the game as well as looking for patterns and relationships can be very beneficial to more easily recognize areas that may be exploited.
- Gaining experience with how player's actions align with their cards or hand strength can be very useful.